Exponents Stretch

1. $256^{1/4} = \sqrt[4]{256} = \sqrt[4]{16 \times 16} = \sqrt[4]{(2 \times 2)^4} = \sqrt[4]{4^4} = \mathbf{4}$.

2. $\dfrac{(x^2y^3)^3}{x^2y^{-2}} = \dfrac{x^6y^9}{x^2y^{-2}} = x^4y^{11} = x^ay^b$. So, $a = 4$, $b = 11$ and $a + b = 4 + 11 = \mathbf{15}$.

3. $2^{3^2} = 2^9 = \mathbf{512}$.

4. $\left(2\frac{1}{4}\right)^{1/2} + \left(2\frac{1}{4}\right)^{-1/2} = \left(\frac{9}{4}\right)^{1/2} + \left(\frac{9}{4}\right)^{-1/2} = \left(\frac{9}{4}\right)^{1/2} + \left(\frac{4}{9}\right)^{1/2} = \sqrt{\frac{9}{4}} + \sqrt{\frac{4}{9}} = \frac{3}{2} + \frac{2}{3} = \frac{9+4}{6} = \frac{13}{6} = \mathbf{2\,\frac{1}{6}}$.

5. $\left(\sqrt{w}\right)^3 = (w^{1/2})^3 = w^{3/2} = w^{m/n}$. So, $m = 3$, $n = 2$ and $m - n = 3 - 2 = \mathbf{1}$.

6. The equation $9^{x-1} = 27$ can be rewritten as $(3^2)^{x-1} = 3^3 \rightarrow 3^{2x-2} = 3^3$. Now that the base of each power is 3, we can set the exponents equal to each other to get $2x - 2 = 3$. Solving for x yields $2x = 5$, and $x = \mathbf{5/2}$.

7. $\dfrac{2^{32} - 2^{29}}{2^{31} - 2^{30}} = \dfrac{2^{29}(2^3 - 1)}{2^{29}(2^2 - 2)} = \dfrac{\cancel{2^{29}}(8-1)}{\cancel{2^{29}}(4-2)} = \mathbf{7/2}$.

8. $2^d = \dfrac{32^7}{16^5} = \dfrac{(2^5)^7}{(2^4)^6} = \dfrac{2^{35}}{2^{24}} = 2^{11}$. So, $d = \mathbf{11}$.

9. The equation $\left(\sqrt{3}\right)^t = \sqrt[9]{9}$ can be rewritten as follows: $3^{t/2} = 9^{1/9} \rightarrow 3^{t/2} = (3^2)^{1/9} \rightarrow 3^{t/2} = 3^{2/9}$. Now that the base of each power is 3, we can set the exponents equal to each other to get $t/2 = 2/9$. Solving for t yields $t = \mathbf{4/9}$.

10. There are three cases in which the expression on the left-hand side of the equation equals 1. Case 1 is when the base $x - 2 = 1$. Case 2 is when the exponent $x^2 + 7x + 12 = 0$. Case 3 is when the base $x - 2 = -1$ and the exponent $x^2 + 7x + 12$ is even. Let's examine each case.

Case 1: When $x - 2 = 1$, $x = \underline{3}$. It doesn't matter what the exponent is because 1 raised to any power is 1.

Case 2: When $x^2 + 7x + 12 = 0$, we can solve the equation by factoring the quadratic. Doing so yields $(x + 3)(x + 4) = 0$. So, this is true when $x = \underline{-3}$ and when $x = \underline{-4}$.

Case 3: When $x - 2 = -1$, $x = 1$. Substituting this for x in the exponent, we get $1^2 + 7(1) + 12 = 1 + 7 + 12 = 20$, which is even and not equal to 0. So, $x = \underline{1}$ is also a solution.

Therefore, the sum of the solutions is $3 + (-3) + (-4) + 1 = \mathbf{-3}$.