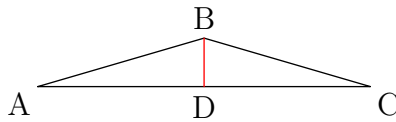


Answer Sheet

Number	Answer	Problem ID
1	168 sq cm	5AAD3
2	1 inches	44A54
3	72 cm	5DA54
4	31 units	CB103
5	45 (cm)	D41D3
6	1 + sqrt3 centimeters	23354
7	1	C2D54
8	1 (sq units)	531D3
9	$100\sqrt{3}$ cm ²	A1A11
10	20.5 square units	2D554
11	$3\sqrt{33}$ (units)	0B253
12	3 units	BB354
13	10	13044
14	15 meters	C4A4D
15	6sqrt3 square inches	35A54
16	5sqrt2 units	D5A44
17	8 cm	BA314
18	18 degrees	2B214
19	1 4/5 units	D1312
20	1/16	15B14
21	6 sq units	13003
22	$640/3$ (sq units)	01553
23	11/32	44314
24	30sqrt2 units	45A44

Solutions

- (1) **168 sq cm** **ID: [5AAD3]**



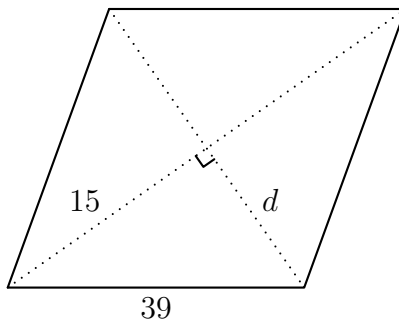
Because ABC is isosceles, BD is perpendicular to AC and it bisects AC . Thus, $AD = \frac{48}{2} = 24$. Now we see ABD is a 7-24-25 right triangle, so $BD = 7$. Calculating the area of ABC , we get $\frac{1}{2} \cdot 48 \cdot 7 = 24 \cdot 7 = \boxed{168}$ square centimeters.

- (2) **1 inches** **ID: [44A54]**

Note that this is a scaled-down version of the oft-used 3 – 4 – 5 right triangle. The other sides were scaled down by a factor of 5, so the hypotenuse has shrunk by the same, and is thus $\boxed{1}$ inch.

- (3) **72 cm** **ID: [5DA54]**

From the information given in the problem, we can construct a diagram, as shown below (not to scale):



Since a rhombus is a parallelogram with four congruent sides, we know that each side must measure $\frac{156 \text{ cm}}{4} = 39 \text{ cm}$. Also, because the diagonals of a rhombus are perpendicular bisectors, we know that one leg of the right triangle shown must be half of the shorter diagonal, or 15 cm. Thus, we can use the Pythagorean theorem to find the other side of the triangle.

$$d = \sqrt{39^2 - 15^2} = 36$$

(We can also scale the right triangle with side length 5, 12, and 13 to get a right triangle with side lengths $3 \cdot 5 = 15$, $3 \cdot 12 = 36$, and $3 \cdot 13 = 39$.) Thus, the longer diagonal is $2d$, or $\boxed{72 \text{ cm}}$.

- (4) **31 units** ID: [CB103]

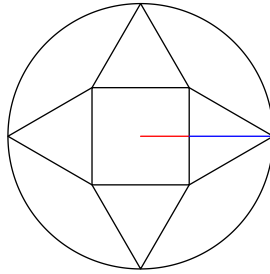
The Triangle Inequality says that the sum of the lengths of any two sides must be greater than the length of the third side. That means $8 + 8 = 16$ must be greater than the length of the third side. The third side has a whole number length, so the greatest possible length is 15 units. That makes the perimeter $8 + 8 + 15 = \boxed{31}$ units.

- (5) **45 (cm)** ID: [D41D3]

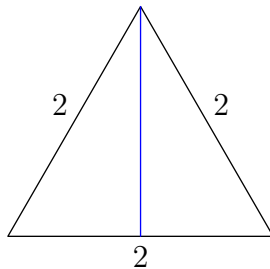
Let the ratio of side lengths between the similar triangle and the given triangle be x , so the lengths of the similar triangle are $6x$, $7x$, and $9x$. We are given that $6x + 7x + 9x = 110$; solving yields $x = \frac{110}{(6+7+9)} = \frac{110}{22} = 5$. The length of the longest side is thus $9x = 9 \cdot 5 = \boxed{45}$.

- (6) **$1 + \sqrt{3}$ centimeters** ID: [23354]

We can think of the circle's radius as being composed of two parts:



The red portion begins from the center of the circle (and the center of the square as well, due to the nature of the construction) and is 1 cm long. The blue portion is slightly trickier.



Note that the blue line divides the equilateral triangle into two $30^\circ - 60^\circ - 90^\circ$ triangles. Thus, the blue line has length $\sqrt{3}$.

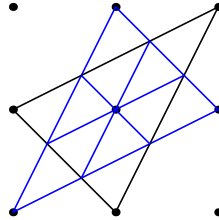
Putting the two parts together, we see that the circle has a radius of $\boxed{1 + \sqrt{3}}$ cm.

(7) **1** ID: [C2D54]

The median of a triangle is a line segment that connects a vertex with the midpoint of the opposite side. Thus, we're looking for the x -coordinate of the midpoint of $\overline{AB}$, which is $\frac{-3+5}{2} = \boxed{1}$.

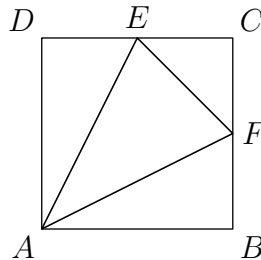
(8) **1 (sq units)** ID: [531D3]

We color one of the triangles blue, and draw three blue segments connecting its points of intersection with the other triangle.



Because of the symmetry inherent in the grid and the two triangles (which are both isosceles), these three blue segments divide the blue triangle into congruent smaller triangles. The blue triangle contains 9 of these congruent smaller triangles.

The region of overlap of the two triangles is a hexagonal region. As per the diagram above, this hexagonal region contains 6 of these congruent smaller triangles. Thus, the area of the hexagonal region is $6/9 = 2/3$ of the area of one of the isosceles triangles. We compute the area of one isosceles triangle as follows:

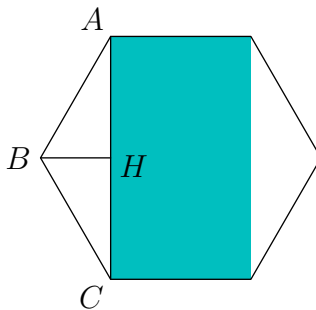


Label points A, B, C, D, E, F as above. To compute the area of this triangle ($\triangle AEF$), notice how it is equal to the area of square $ABCD$ minus the areas of triangles $\triangle ADE$, $\triangle ABF$, and $\triangle ECF$. The square has side length 2 units, so the area of $\triangle ADE$ and $\triangle ABF$ is $\frac{1}{2}(2)(1) = 1$ and the area of $\triangle ECF$ is $\frac{1}{2}(1)(1) = \frac{1}{2}$. The area of square $ABCD$ is $2^2 = 4$, so the area of triangle $\triangle AEF$ is equal to $4 - 2(1) - \frac{1}{2} = \frac{3}{2}$.

Finally, remember that the hexagonal region has area $2/3$ of the area of the triangle, or $\frac{2}{3} \cdot \frac{3}{2} = 1$. Thus, the answer is $\boxed{1}$.

- (9) $100\sqrt{3} \text{ cm}^2$ ID: [A1A11]

Label points A , B , C as shown below, and let H be the foot of the perpendicular from B to AC .



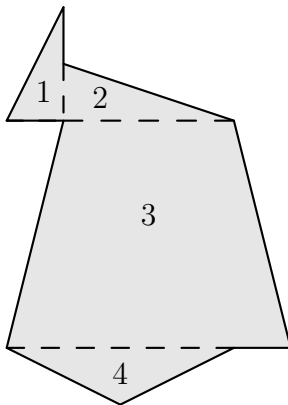
Since the hexagon is regular, $\angle ABC = 120^\circ$ and $\angle ABH = \angle CBH = 120^\circ/2 = 60^\circ$. Thus, $\triangle ABH$ and $\triangle CBH$ are congruent $30^\circ - 60^\circ - 90^\circ$ triangles. These triangles are each half an equilateral triangle, so their short leg is half as long as their hypotenuse.

Since $AB = BC = 10$, we have $BH = AB/2 = 5$ and $AH = CH = \sqrt{10^2 - 5^2} = \sqrt{75} = 5\sqrt{3}$. (Notice that this value is $\sqrt{3}$ times the length of BH , the short leg. In general, the ratio of the sides in a $30^\circ - 60^\circ - 90^\circ$ is $1 : \sqrt{3} : 2$, which can be shown by the Pythagorean Theorem.) Then, $AC = 2 \cdot 5\sqrt{3} = 10\sqrt{3}$.

The shaded region is a rectangle with base length 10 and height length $10\sqrt{3}$; its area is $10 \cdot 10\sqrt{3} = \boxed{100\sqrt{3}}$ square cm.

- (10) 20.5 square units ID: [2D554]

Note that we can separate the figure into 4 distinct shapes.

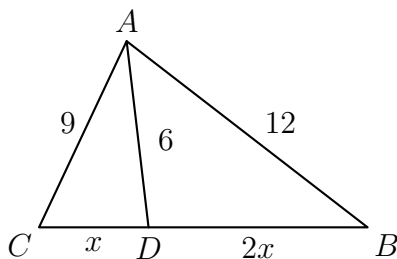


Section 1 is a triangle with an area of $\frac{1}{2}(1)(2) = 1 \text{ un}^2$. Section 2 is also a triangle with an area of $\frac{1}{2}(3)(1) = 1.5 \text{ un}^2$.

Section 3 is an isosceles trapezoid, with an area of $\frac{4(5+3)}{2} = 16 \text{ un}^2$. Finally, section 4 is the triangle at the bottom, with an area of $\frac{1}{2}(4)(1) = 2 \text{ un}^2$. Thus, the total area of the shaded figure is

$$1 + 1.5 + 16 + 2 = \boxed{20.5} \text{ un}^2$$

(11) $3\sqrt{33}$ (units) ID: [0B253]



We use Stewart's on $\triangle ABC$:

$$9^2(2x) + 12^2(x) = 6^2(3x) + (2x)(x)(3x).$$

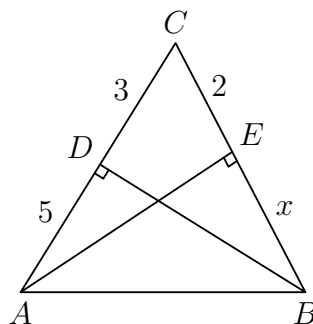
Simplifying yields $198x = 6x^3$; the nonzero, positive solution is $x = \sqrt{33}$. It follows that $BC = 3x = \boxed{3\sqrt{33}}$.

(12) **3 units** ID: [BB354]

If two triangles are similar, their corresponding sides are in the ratio $a : b$, and their areas are in the ratio $a^2 : b^2$.

$\triangle XBY$ and $\triangle ABC$ are both equilateral and thus are similar by AAA. If their area ratio is $\frac{1}{4}$, then the ratio of their corresponding side lengths is the square root of the area ratio: $\sqrt{\frac{1}{4}} = \frac{1}{2}$. We then have the ratio $\frac{XY}{AC} = \frac{1}{2}$. Since we are given that $AC = 6$, then $\frac{XY}{6} = \frac{1}{2}$, so $XY = \boxed{3}$ units.

- (13) **10** **ID: [13044]**
 Let us label this diagram.



$\triangle ACE$ and $\triangle BCD$ are similar by AA since they share $\angle ACB$ and $\angle AEC$ and $\angle BDC$ are both right angles and hence congruent. So

$$\frac{CE}{CD} = \frac{AC}{BC}.$$

Plugging in values, we have

$$\frac{2}{3} = \frac{8}{x+2}.$$

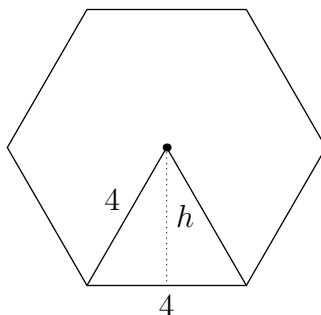
Solving this gives $x + 2 = 12$, or $x = \boxed{10}$.

- (14) **15 meters** **ID: [C4A4D]**

Since the square has area 9 m^2 , its side length is $\sqrt{9} = 3$ meters. Since the triangle is equilateral, all 5 sides of the pentagon have length 3 meters, so its perimeter is $3 \cdot 5 = \boxed{15 \text{ meters}}$.

- (15) **6sqrt3 square inches** ID: [35A54]

Regardless of the various ways we could select these four triangles, their total area is equal to the area of the hexagon that contains them.



We can split the hexagon into 6 equilateral triangles to find the area, as shown above. Thus, the total area A of the hexagon is

$$A = 6 \left(\frac{1}{2} 4h \right)$$

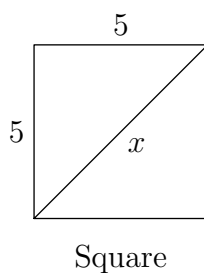
$$A = 12h$$

We can form a right triangle with side lengths $2, h$ and hypotenuse 4 . Using the Pythagorean theorem, we see that $2^2 + h^2 = 4^2$, so $h = \sqrt{12} = 2\sqrt{3}$. Thus, $A = 12(2\sqrt{3}) = 24\sqrt{3}$.

Thus, the average of the 4 triangles is $\frac{A}{4}$, or $\boxed{6\sqrt{3}}$ square inches.

- (16) **5sqrt2 units** ID: [D5A44]

Let's label another side of the square, since all of the sides are equal:



Using the Pythagorean for the triangle formed by the diagonal, we know that

$$5^2 + 5^2 = x^2$$

$$25 + 25 = x^2$$

$$x^2 = 50$$

$$x = \boxed{5\sqrt{2}}$$

- (17) **8 cm** ID: [BA314]

Since YW is an altitude, $\triangle WXY$ is a right triangle. This means that we can use the Pythagorean Theorem. We have

$$\begin{aligned}(XY)^2 &= (YW)^2 + (XW)^2 \\ 17^2 &= (YW)^2 + 15^2 \\ 289 &= (YW)^2 + 225 \\ (YW)^2 &= 64 \\ YW &= 8.\end{aligned}$$

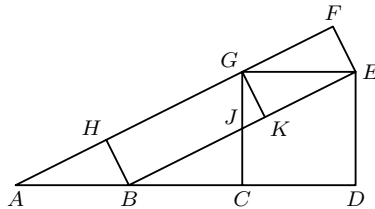
Therefore, the length of altitude YW is 8 cm.

- (18) **18 degrees** ID: [2B214]

Two triangles are similar if the measures of corresponding sides are all in the same proportion. Since the measures of the sides of the new triangle are each twice as long as the measures of the corresponding sides of the original triangle, the two triangles in this problem are similar. Similar triangles also have all corresponding angles congruent. Therefore, if the smallest angle of the original triangle measures 18 degrees, the smallest angle of the new triangle also measures 18 degrees.

- (19) **1 4/5 units** ID: [D1312]

Let J be the intersection of $\overline{BE}$ and $\overline{GC}$.



Observe that $BD = \sqrt{BE^2 - DE^2} = \sqrt{5^2 - 3^2} = 4$ units. By the similarity of triangles BCJ and BDE , we have

$$\frac{CJ}{BC} = \frac{DE}{BD},$$

which after substituting becomes

$$\frac{CJ}{4-3} = \frac{3}{4}.$$

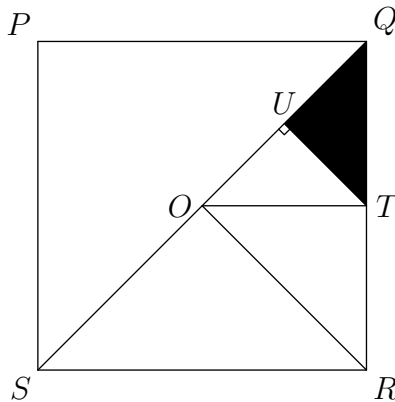
We solve to find $CJ = \frac{3}{4}$, which implies that $GJ = 3 - \frac{3}{4} = \frac{9}{4}$. Applying the Pythagorean theorem to triangle GJE , we find $EJ = \sqrt{3^2 + \left(\frac{9}{4}\right)^2} = \frac{15}{4}$. Define K to be the foot of the perpendicular from G to side EJ . By the similarity of triangles GKJ and EGJ , we have

$$\frac{GK}{GJ} = \frac{EG}{EJ} \implies \frac{GK}{\frac{9}{4}} = \frac{3}{\frac{15}{4}},$$

which we solve to find $GK = \frac{9}{5}$. Since $GKBH$ is a rectangle, $BH = GK = \frac{9}{5} = \boxed{1\frac{4}{5}}$ units.

(20) **1/16** ID: [15B14]

In order to find the relationship between $\triangle QUT$ and square $PQRS$, let us find O - the midpoint of segment SQ , and draw the segments connecting it to R and T . We have to split our square up into pieces, and we need to figure out which part of the square each piece makes:



Since SQ is a diagonal, we know that it splits the square into two equal parts. Now, $\triangle QRS$ is isosceles, since two of its sides are sides of the original square. This means that its median (which is also the altitude) splits it into halves, so $\triangle QRO$ is half of $\triangle QRS$, and therefore a quarter of the original square (in area). Furthermore, in a right triangle, the altitude to the hypotenuse is half the length of the hypotenuse, so $OR \cong OQ$, and $\triangle QRO$ is also isosceles. Much like before, OT is the median (and altitude) of $\triangle QRO$, so it splits it into halves, and $\triangle QOT$ is therefore a eighth of the area of the original square. Repeating the same logic once again, $\triangle QOT$ is isosceles for the same reasons as before, so UT must split it into halves as well. Thus, $\triangle QUT$ is $\boxed{1/16}$ of the area of square $PQRS$.

(21) **6 sq units** ID: [13003]

Since $5 > 4$, 4 cannot be the length of the hypotenuse. Thus either 4 and 5 are the lengths of the two smaller sides, or 5 is the hypotenuse, meaning the two smaller sides are 4 and 3. In this latter case, the area will be smaller, so the area is $\frac{(3)(4)}{2} = \boxed{6}$.

(22) **640/3 (sq units)** ID: [01553]

Let the shortest side have length $5x$. Then we have $5x + 12x + 13x = 80$, so $x = 80/30 = 8/3$. It follows that the area is $(1/2)(5x)(12x) = 30x^2 = \boxed{640/3}$ square units.

(23) **11/32** ID: [44314]

Without loss of generality we can let each of edges of square WXYZ have a length of $4x$. Then, the area of square WXYZ is $(4x)^2 = 16x^2$. The area of each of the smaller congruent squares is one fourth of this, or $4x^2$.

The area of the gray part of the smaller square in the X corner is one half of the area of the smaller square, or $2x^2$.

The smaller square in the W corner is made up of a gray quadrilateral and four white triangles. Each white triangle has base and height each of length x . The area of the gray part is equal to the area of smaller square minus the total area of the four triangles, or $4x^2 - 4\left(\frac{1}{2} \cdot x \cdot x\right) = 4x^2 - 2x^2 = 2x^2$.

The gray area in the smaller square in the Z corner is a square with side length x , so it has an area of x^2 .

The gray area in the smaller square in the Y corner has the same area as one of the white triangles in the smaller square in the W corner, or $\frac{1}{2} \cdot x \cdot x = \frac{1}{2}x^2$.

The sum of the areas of the gray parts is $2x^2 + 2x^2 + x^2 + \frac{1}{2}x^2 = \frac{11}{2}x^2$. Therefore, the fractional part of square

WXYZ that is gray is $\frac{\frac{11}{2}x^2}{16x^2} = \frac{\frac{11}{2}}{16} = \boxed{\frac{11}{32}}$.

(24) **30sqrt2 units** ID: [45A44]

We can consider AH to be the hypotenuse of $\triangle ADH$. Before using the Pythagorean theorem to find x , we will have to apply it to $\triangle EDH$ to find the length of another diagonal, DH .

$$ED^2 + EH^2 = DH^2$$

$$30^2 + 24^2 = DH^2$$

$$900 + 576 = DH^2$$

$$DH^2 = 1476$$

$$DH = \sqrt{1476} = 6\sqrt{41}$$

Thus now have enough information to find x ,

$$AD^2 + DH^2 = AH^2$$

$$18^2 + 1476 = x^2$$

$$x^2 = 1800$$

$$x = \boxed{30\sqrt{2}}$$