



Divisor Counting Stretch

The number 144 has 15 **divisors**. One way to see this is by *brute force* – that is, simply list all of the divisors: 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144.

A more elegant way to get the answer is to note that $144 = 2^4 \times 3^2$, and that every divisor of 144 is some combination of 2's and 3's, since these are the only **prime numbers** in the prime factorization of 144.

When building a divisor of 144, we have 5 choices of how many 2's to include: we may include 4, 3, 2, 1 or 0. Similarly, we have 3 choices of how many 3's to include in our divisor: 2, 1 or 0. That gives us a total of $3 \times 5 = 15$ divisors. You may wonder, "*but if we include zero 2's and zero 3's in our divisor... what divisor have we built?*" The answer is that we have built $2^0 \times 3^0 = 1$, which is indeed a divisor of 144!

Note: In many contexts, the term "divisor" is assumed to mean specifically "positive integer divisor"; in other contexts, including MATHCOUNTS competition materials, negative integers qualify as divisors unless explicitly noted otherwise. Explicit notice is hereby given that only positive integers are to be regarded as divisors for all problems in this Stretch.

1. _____ divisors How many divisors does 6000 have?
2. _____ What is the smallest positive integer that has the same number of divisors as the integer 6000?
3. _____ What is the smallest positive integer with 13 divisors?
4. _____ What is the smallest positive integer with 100 divisors?
5. _____ divisors How many odd divisors does the number 450 have?
6. _____ A divisor of 7200 is chosen. What is the probability that it is a multiple of 6? Express your answer as a common fraction.
7. _____ What is the smallest positive integer with 12 divisors such that exactly 6 of those divisors are even and exactly 6 are multiples of 3?
8. _____ What is the units digit of the product of all of the divisors of 72?
9. _____ What is the positive difference between the two smallest positive integers that each have 10 divisors?
10. _____ Delilah chooses a divisor of 126,000 at random. What is the probability that she chooses a multiple of 20? Express your answer as a common fraction.