

Combinations and Permutations Stretch

11. If we are picking 5 pieces of candy from 8 different pieces of candy, there are 8 ways to choose the first, 7 ways to choose the second, 6 ways to choose the third, 5 ways to choose the fourth and 4 ways to choose the fifth. That gives us $8 \times 7 \times 6 \times 5 \times 4 = 6720$ ways. However, for each way of picking 5 pieces of candy, there are $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ ways to order them. Each group of 5 candies has 120 orders. Since order does not matter in combinations, there are $6720/120 = 56$ combinations. Alternatively, we can use the formula for combinations to get ${}_8C_5 = 8!/(5! \times 3!) = (8 \times 7 \times 6)/(3 \times 2 \times 1) = 8 \times 7 = 56$ combinations.

12. The value of "8 choose 6" is ${}_8C_6 = 8!/(6! \times 2!) = (8 \times 7)/(2 \times 1) = 28$.

13. When flipping 4 coins, the number of heads can be 0, 1, 2, 3 or 4 so there are 5 combinations. (These are the same combinations as having 4, 3, 2, 1 or 0 tails.)

14. In permutations, order does matter. Each die rolled can show 1 through 6 on the top face, so the total number of permutations equals $6 \times 6 \times 6 = 6^3 = 216$ permutations.

15. Juan is picking 4 ingredients from 7, so the number of combinations is $7!/(4! \times 3!) = (7 \times 6 \times 5)/(3 \times 2 \times 1) = 7 \times 5 = 35$ combinations.

16. If the marbles were all different colors, the number of orders (permutations) would be $7!$. In this problem, 3 groups of marbles have the same color. Because the different orders of choosing the same color marbles are not distinguishable, they must be divided out. So the number of color orders in this problem is then $7!/(3! \times 2! \times 2!) = (7 \times 6 \times 5 \times 4)/(2 \times 1 \times 2 \times 1) = 7 \times 6 \times 5 = 210$ color orders.

17. If the bouquet must include 1 of each flower, 4 of the 6 flowers must include all 4 types. There is only 1 such combination. This leaves two flowers to choose to finish the bouquet, which can be either the same or different. There are 4 ways for the remaining two flowers to be the same and $C(4, 2) = 4!/(2! \times 2!) = (4 \times 3)/(2 \times 1) = 6$ ways for the remaining two flowers to be different. So the total number of combinations is $1 \times (4 + 6) = 10$ combinations.

18. First, 2 of the 6 students are chosen to fill the 2 positions. There are $C(6, 2) = 6!/(2! \times 4!) = (6 \times 5)/(2 \times 1) = 15$ ways to do this. For each of the 2 students chosen, there are 2 ways to assign them the 2 different positions, so this makes a total of $15 \times 2 = 30$ combinations.

19. There are 3 combinations in which all the cards are the same (1 combination for each color), and there is 1 combination in which all the cards are different colors. There are $3 \times 2 = 6$ combinations in which 2 of the cards are the same and 1 is different (3 ways to pick the color for the 2 cards that are the same and 2 ways to pick the other color). This gives $3 + 1 + 6 = 10$ combinations.

20. In this problem, each of the 3 cards can have 1 of 3 different colors, so the number of permutations equals $3 \times 3 \times 3 = 3^3 = 27$ permutations.