

Team 1

$A = 4P$; $100 = A + P = 5P$; $P = 100/5 = \mathbf{20}$.

Team 2

There is a total of $8 + 3 + 7 + 10 = 28$ students, of which 7 are wearing both long pants and a jacket, so the desired probability is $8/28 = \mathbf{2/7}$.

Team 3

Selling 60 kazoos requires buying $60/4 = 15$ packs at \$3 per pack for a total of \$45 spent. Then they are resold as 20 sets of 3 and \$2.85 per set for a total of \$57 revenue. The profit is $\$57 - \$45 = \mathbf{\$12}$ or $\mathbf{\$12.00}$.

Team 4

Let d be the number of doughnuts bought by Ashton. Then $24 = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)d = d/3$, so $d = 3 \times 24 = \mathbf{72}$.

Team 5

There are 3 rows, 3 columns, and 2 diagonals for a total of 8 qualifying arrangements out of a total of $\binom{9}{3} = \frac{9 \times 8 \times 7}{3 \times 2 \times 1} = 12 \times 7 = 84$ possible arrangements (ordering of selection of the three squares is unimportant and not accounted for in both cases). Therefore, the probability is $\frac{8}{84} = \frac{2}{21}$.

Team 6

$D = 2B = 3M$; $B = M + 8$. Therefore, $3M = D = 2B = 2(M + 8) = 2M + 16$. Subtracting $2M$ from both ends yields $M = 16$, $D = 3M = 3 \times 16 = \mathbf{48}$.

Team 7

Start with the greatest possible square and work downward decomposing squares into sums of smaller squares. For the following list, in the interest of space and readability, when a particular perfect square occurs at least 3 times, it is denoted as a product of the count times the square:

[1] $4^2 + 1^2 + 1^2$; [2] $3^2 + 3^2$; [3] $3^2 + 2^2 + 2^2 + 1^2$; [4] $3^2 + 2^2 + 5(1^2)$; [5] $3^2 + 9(1^2)$; [6] $4(2^2) + 1^2 + 1^2$; [7] $3(2^2) + 6(1^2)$; [8] $2^2 + 2^2 + 10(1^2)$; [9] $2^2 + 14(1^2)$; [10] $18(1^2)$.

There are **10** ways.

Team 8

The sum of the terms of a finite arithmetic sequence is the product of the number of terms times the average of the first and last terms. There are 100 terms, the first of which is given to be 32. We are also given that the second term is 35, so the common difference is $35 - 32 = 3$. The last term is term #100, which is 99 after term #1, so the last term is $32 + 99 \times 3 = 329$. Therefore, the sum is

$$100 \times \frac{32+329}{2} = 100 \times \frac{361}{2} = \mathbf{18\,050}.$$

Team 9

Let p , n , and d be the number of pennies, nickels, and dimes, respectively. We are given $d = n + 3$, $n = p + 6$, so $d = p + 9$. The total number of coins is $p + n + d = p + (p + 6) + (p + 9) = 3p + 15$. We need the total value of the coins. The value of each penny, nickel, and dime is 1 ¢, 5 ¢, and 10 ¢, respectively. The total value is $p(1 \text{ ¢}) + n(5 \text{ ¢}) + d(10 \text{ ¢}) = p(1 \text{ ¢}) + (p + 6)(5 \text{ ¢}) + (p + 9)(10 \text{ ¢}) = 16p \text{ ¢} + 120 \text{ ¢}$. The average coin value is 7 ¢, so the total value is $(3p + 15)(7 \text{ ¢}) = 21p \text{ ¢} + 105 \text{ ¢}$. The two expressions of the total value must be equal, so we have $16p \text{ ¢} + 120 \text{ ¢} = 21p \text{ ¢} + 105 \text{ ¢}$. Each term in the equation has a common factor ¢, so divide both sides of the equation by ¢. Subtracting $16p + 105$ from both sides yields $15 = 5p$, so $p = 15/5 = 3$ and $d = p + 9 = \mathbf{12}$.

Team 10

Set up an xy -coordinate plane with the x -axis along $\overline{AC}$ and the y -axis along $\overline{DB}$, so that the origin O is at the midpoint of $\overline{AC}$ and the four vertices in question are $A(-12, 0)$, $C(12, 0)$, $B(0, 16)$, and $D(0, 9)$ —with $OD = 9$ deriving from CDO being a 3-4-5 right triangle with scale factor 3, and $OB = 16$ deriving from CBO being a 3-4-5 right triangle with scale factor 4. The point of minimum sum must be on the y -axis, because moving to the right from the y -axis causes PA to increase slightly faster than PC decreases, plus PB and PD increase as well; symmetry says the case of P moving left of the y -axis behaves likewise. If P starts at O and moves toward D , each unit of movement causes $PB + PD$ to decrease by 2 while $PA + PC$ increases much less, for a net decrease in the sum. Once P passes D moving toward B , PB decreases at the same rate that PD increases, but PA and PC are increasing. Therefore, D is the optimum position, resulting in $DA + DB + DC + DD = 15 + 7 + 15 + 0 = \mathbf{37}$.