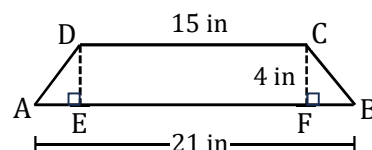


**Target 1**

The integers 1 through 10 sum to  $\frac{10(10+1)}{2} = 55$ . Dropping one value would yield a value for the sum in the interval 45 through 54, of which the only perfect square is  $7^2 = 49$ , so  $55 - 49 = 6$ .

**Target 2**

Because the trapezoid is isosceles,  $\overline{AD} \cong \overline{BC}$ . Because  $\overline{DE}$  and  $\overline{CF}$  are heights of the trapezoid,  $\overline{DE} \cong \overline{CF}$ . Therefore, by the Hypotenuse-Leg theorem,  $\triangle DAE \cong \triangle CBF$ , so corresponding sides  $\overline{AE}$  and  $\overline{BF}$  are congruent. As opposite sides of a rectangle,  $EF = DC = 15$  in.



$AE + BF = 21$  in  $- EF = 6$  in, so  $AE = BF = 3$  in. Thus right triangles

$\triangle DAE$  and  $\triangle CBF$  are 3-4-5 triangles with scale factor 3 in and  $AD = BC = 5$  in. Therefore, the perimeter is  $21$  in  $+ 15$  in  $+ 2 \times 5$  in  $= 46$  in.

**Target 3**

Line 1 passes through  $(2, 5)$  and  $(0, 7)$ , so its slope is  $m_1 = \frac{7-5}{0-2} = -1$ .

The slope of perpendicular line 2 is  $m_2 = -\frac{1}{m_1} = -\frac{1}{-1} = 1$ .

Therefore, line 2 passes through  $(2, 5)$  and  $(0, b_2)$  for  $b_2 = 5 + (0 - 2)1 = 3$ .

- [NOTE: There was some controversy over the form of the answer at some of the competitions. There are two different definitions of y-intercept in common use:
  - (1) the y-value of a line or curve when  $x = 0$ ;
  - (2) the coordinates of a point, as an ordered pair, where a line or curve intersects the y-axis.
 In this question, the y-intercept would be stated as 3 per definition (1) and as  $(0, 3)$  per definition (2). Some students wrote the latter as the answer—likely because a teacher taught them vehemently and vigorously that an intercept is always and only a point, which, therefore, *must* be expressed as an ordered pair with *both* components explicitly stated no matter what. Different mathematicians working in different branches of mathematics often find it convenient and useful to borrow a term from another branch but tweak the definition to suit their own special purposes; there is no authoritative organization in mathematics to remedy such differences. It is appropriate for a teacher to state the specific definitions that will be adhered to in class, likely subject to deduction of points if violated in classwork; however, students should also be made aware that there are people who use alternative definitions that are not wrong

outside that class, and students will likely encounter some of them, so a person needs to be flexible when reading mathematics material. In the case of this question, there are three very strong indicators—two within the question itself and one in the School Handbook—that the correct answer is 3, not  $(0, 3)$ :

(1) The question states “the  $y$ -intercept of one of the lines is 7”, thus providing an explicit example of what is being looked for, so follow the same pattern for the answer, even if that pattern does not match what you were taught. The instructions in a question prevail.

(2) With the one exception of monetary amounts, the default is that all answers are integers unless explicitly stated to the contrary (Express your answer in simplest radical form. Express your answer as a common fraction. ...). An ordered pair of integers does not qualify as an integer, so if the answer was to be given as an ordered pair, then that would have been explicitly stated, but it was not.

(3), In this year’s School Handbook on page 25 in the lower right corner, there is a table “Equation of a Line” and the second entry is “Slope-Intercept Form” with equation “ $y = mx + b$ ”, where “ $m = \text{slope}$   $b = y\text{-intercept}$ ”, so that the real number  $b$ , thus not the coordinate  $(0, b)$ , is explicitly stated to be the  $y$ -intercept.

Therefore, the directive to scorers at the concerned competitions was to regard  $(0, 3)$  as an incorrect answer.

#### **Target 4**

The only ways to add distinct prime numbers to yield 10 are  $2 + 3 + 5$  and  $3 + 7$ . For products of these sets of primes to be less than 100, we have  $2 \times 3 \times 5 = 30$ ;  $2^2 \times 3 \times 5 = 60$ ;  $2 \times 3^2 \times 5 = 90$ ;  $3 \times 7 = 21$ ;  $3^2 \times 7 = 63$ . The sum is  $30 + 60 + 90 + 21 + 63 = \mathbf{264}$ .

#### **Target 5**

By subtracting 2 from the various inequalities,  $-6 \leq x + 2 \leq 6$  is equivalent to  $-8 \leq x \leq 4$ , which has  $4 - (-8) + 1 = \mathbf{13}$  integer solutions.

#### **Target 6**

The two moduli, 5 and 7, are small, so brute force works faster than a high-powered technique like Chinese remainder theorem. To step through candidates faster, focus on the greater number 7. The least positive integer of the form  $7k + 1$  is 7 (for  $k = 0$ ). That is not a multiple of 5. Increment by 7: 8 (No); 15 (Yes). With the moduli 5 and 7 being relatively prime, the next qualifying values will occur in steps of  $5 \times 7 = 35$ , so the first three are 15; 50; 85. This is an arithmetic sequence of terms whose sum is the product of the count of terms times the median value:  $3 \times 50 = \mathbf{150}$ .

#### **Target 7**

Designate the  $n$ th triangular number as  $T_n$ , with  $T_n = 1 + 2 + 3 + \cdots + n = n(n + 1)/2$  for any positive integer  $n$ . Thus,  $T_8$  is the sum of the integers 1 through 8. Subtracting  $T_6$  means subtracting the integers 1 through 6, which are the first 6 terms of  $T_8$ , leaving only  $7 + 8 = \mathbf{15}$ .

**Target 8**

Because the triangle is specified to be equilateral, rotating line segment  $\overline{BA}$  by  $+60^\circ$  about B would result in a match with  $\overline{BC}$ . Drop an altitude from C meeting  $\overline{BA}$  at midpoint D.  $\triangle CBD$  is a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle.

Going from B to D involves moving  $a/2$  right and  $b/2$  down.

$DC = \sqrt{3} BD$  and  $\overline{DC} \perp \overline{BD}$ , so the motion from D to C involves

multiplying the motions from B to D by  $\sqrt{3}$ , and, because the slopes of two perpendicular lines are negative reciprocals of each other,

swapping which value is horizontal and vertical, and negating one of the two motions so that we are

moving up and right. Thus, point A would be transformed to  $\left(\frac{1}{2}a + \frac{\sqrt{3}}{2}b, \frac{1}{2}b + \frac{\sqrt{3}}{2}a\right) = C = (20, 25)$ .

Therefore, we have a system of two linear equations in the two unknowns  $a$  and  $b$ .

$\frac{1}{2}a + \frac{\sqrt{3}}{2}b = 20$ , which can be simplified a bit to Eqn ①  $a + \sqrt{3}b = 40$ .

$\frac{\sqrt{3}}{2}a + \frac{1}{2}b = 25$ , which can be simplified a bit to Eqn ②  $\sqrt{3}a + b = 50$ .

$\sqrt{3}$ ① - ② yields  $2b = 40\sqrt{3} - 50$ , and  $\sqrt{3}$ ② - ① yields  $2a = 50\sqrt{3} - 40$ . Therefore,

$a = 25\sqrt{3} - 20$  and  $b = 20\sqrt{3} - 25$ . The area of an equilateral triangle with side length  $s$  is  $\frac{\sqrt{3}s^2}{4}$ .

Now,  $s^2 = (AB)^2 = a^2 + b^2 = (25\sqrt{3} - 20)^2 + (20\sqrt{3} - 25)^2$

$= (1875 - 1000\sqrt{3} + 400) + (1200 - 1000\sqrt{3} + 625) = 4100 - 2000\sqrt{3}$ , so the triangular area is

$\frac{\sqrt{3}(4100 - 2000\sqrt{3})}{4} = \frac{(4100\sqrt{3} - 6000)}{4} = 1025\sqrt{3} - 1500 = 275.352 \dots$ , which rounds to the nearest

hundredth as **275.35**.

