

The MATHCOUNTS Bible According to Mr. Diaz

What you must memorize, without excuses and for the rest of your lives
(not just for MATHCOUNTS)

- I. **Squares and square roots:** From 1^2 to 30^2 .
- II. **Cubes and cubic roots:** From 1^3 to 12^3 .
- III. **Powers of 2:** From 2^1 to 2^{12} .
- IV. **Prime numbers from 2 to 109:** It also helps to know the primes in the 100's, like 113, 127, 131, ... It's important to know not just the primes, but *why* 51, 87, 91, and others are *not primes*.
- V. **Sum of the numbers in an arithmetic series:** In an *arithmetic series* the difference between terms is a constant. Example: $4 + 10 + 16 + 22 + \dots + 100$ is an arithmetic series. The formula for the sum is

$$n(a + z) / 2$$

where n is the number of terms in the sequence, a is the lowest term, and z is the highest term. Finding the sum of the above sequence:

$$17(4 + 100) / 2 = 17 * 104 / 2 = 884$$

Why is n equal to 17? Figure it out.

- VI. **Triangle or triangular numbers:** 3, 6, 10, 15, 21, 28, 36, 45, 55 are **triangle** numbers. (This is based on V. above!)

$$n(n + 1) / 2$$

To find $1 + 2 + 3 + 4 + 5 + \dots + n$, just take n , multiply it by its higher consecutive number, and divide by 2.

Example: find $1+2+3+4+5+\dots+28$.

$$28(28 + 1) / 2 = 406$$

- VII. **Pythagorean Theorem:** Applications of this famous relationship occur very often in math competition and on the S.A.T.

$$a^2 + b^2 = c^2$$

- A. **Pythagorean Triples:** Integral values of a, b, and c, where a, b, and c are relatively prime:

3 - 4 - 5 (the most common)

5 - 12 - 13

7 - 24 - 25

9 - 40 - 41

8 - 15 - 17

20 - 21 - 29

12 - 35 - 37

- B. **The 45° - 45° - 90° right triangle, or *right isosceles triangle*:** This is half of a square, where the legs are congruent. If the leg is s, then the *hypotenuse* is $s * \sqrt{2}$ (s times the square root of 2). Example: A square has a perimeter of 10, and you need to know the length of the diagonal.

$$s = 2.5, \text{ so } d = 2.5 * \sqrt{2}.$$

- C. **The 30° - 60° - 90° right triangle:** This is half of an *equilateral* triangle. The short leg, the one opposite the 30° angle, is s, the hypotenuse is 2s, and the long leg, which is opposite the 60° angle, is $s * \sqrt{3}$ (s times the square root of 3).

- VIII. **Number of diagonals in an s-sided polygon:** I've seen so many different applications of this formula:

$$s(s - 3) / 2$$

where s is the number of sides of the polygon. A polygon having 45 sides has $45 * (45 - 3) / 2 = 945$ diagonals.

- IX. **Fraction, decimal, percent equivalencies:** You must know these backward, forward, and upside down. The *halves*, *thirds*, *fourths*, *fifths*, *sixths*, *sevenths*, (yes, *sevenths*!), *eighths*, *ninths*, *tenths*, (so hard, he?), *elevenths*, *twentieths*, *twenty-fifths*, *fiftieths*. It also helps to know the *twelfths*, *fifteenths*, and *sixteenths*. You should, for example, be able to recognize, instantly and without hesitation, that $83 \frac{1}{3}\%$ is $\frac{5}{6}$, and that $\frac{9}{11}$ is $81 \frac{9}{11}\%$.

- X. **Space diagonal of a cube:**

$$s * \sqrt{3}$$

where s is the edge of the cube. This is an application of the Pythagorean Theorem: See Section VII(B) above. Figure out why this is so. Don't expect me to do it for you.

XI. Area and Volume:

- A. Area of a square, given the side: $A = s^2$
- B. Area of a square, given diagonal: $A = d^2/2$
- C. Area of a rhombus, given diagonals: $A = (d_1 d_2)/2$
(B and C are closely related. How?)
- D. Area of triangle: $A = (bh) / 2$
- E. Area of circle: $A = \pi r^2$
- F. Area of trapezoid: $A = 1/2 h (b_1 + b_2)$
- G. Volume of cylinder and prism: $V = B h$
- H. Volume of cone and pyramid: $V = 1/3 (B h)$
- I. Volume of a sphere: $V = 4/3 \pi r^3$
- J. Surface area of a sphere: $A = 4 \pi r^2$

XII. I also expect you to know the following procedures:

- A. *Scientific notation*, both multiplying and dividing numbers written in this form. All you do is apply the rules you've learned about *exponents*.
- B. Turning a *repeating decimal* into a *simple fraction*. You see this almost every week; isn't it time to learn the shortcut for this, once and for all?
- C. Turning a *fractional percent* into a simple fraction. Example: $20 \frac{5}{6}\% = \frac{5}{24}$
- D. Setting up *probability problems*. This is usually plain, simple reading. Know the terms "with replacement"; "without replacement", "at least one".
- E. Be able to generate *Pascal's Triangle* on the spot. There are so many applications of this in *combinations* and *probability*.
- F. Can you think of any more? I can. You should.

This is just the beginning. If you think you can't memorize the relationships and formulas in this "Bible", you are absolutely right. Chances are the person ranked above you knows it better than you do. If, on the other hand, you think this can be done, you're quite right!

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