COMPETITION COACH TOOLKIT

This is a collection of lists, formulas and terms that Mathletes frequently use to solve problems like those found in this handbook. There are many others we could have included, but we hope you find this collection to be a useful reference.

Fraction	Decimal	Percent
$\frac{1}{2}$	0.5	50
$\frac{1}{3}$	$0.\overline{3}$	$33.\overline{3}$
$\frac{1}{4}$	0.25	25
$\frac{1}{5}$	0.2	20
$\frac{1}{6}$	$0.1\overline{6}$	$16.\overline{6}$
$\frac{1}{8}$	0.125	12.5
$\frac{1}{9}$	$0.\overline{1}$	$11.\overline{1}$
$\frac{1}{10}$	0.1	10
$\frac{1}{11}$	$0.\overline{09}$	$9.\overline{09}$
$\frac{1}{12}$	$0.08\overline{3}$	$8.\overline{3}$

n	n^2	n^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000
11	121	1331
12	144	1728
13	169	2197
14	196	2744
15	225	3375

Common Arithmetic Series

$$1 + 2 + 3 + 4 + \cdots + n = \frac{n(n+1)}{2}$$

$$1 + 3 + 5 + 7 + \cdots + (2n-1) = n^2$$

$$2 + 4 + 6 + 8 + \cdots + 2n = n^2 + n$$

Combinations & Permutations

$${}_nC_r = \frac{n!}{r!(n-r)!} \qquad {}_nP_r = \frac{n!}{(n-r)!}$$

Prime Numbers

2	43
3	47
5	53
7	59
11	61
13	67
17	71
19	73
23	79
29	83
31	89
37	97
41	

Sequences & Series

For an **arithmetic sequence** with common difference d:

$$a_n = a_1 + (n-1)d$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

For a **geometric sequence** with common ratio r:

$$a_n = a_1 r^{(n-1)}$$

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right) \quad \text{for } r \neq 1$$

Divisibility Rules

2: units digit is 0, 2, 4, 6 or 8

3: sum of digits is divisible by 3

4: two-digit number formed by tens and units digits is divisible by 4

5: units digit is 0 or 5

6: number is divisible by both 2 and 3

8: three-digit number formed by hundreds, tens and units digits is divisible by 8

9: sum of digits is divisible by 9

10: units digit is 0

Difference of Squares

$$a^2 - b^2 = (a+b)(a-b)$$

Distance Traveled

Distance = Rate × Time

Geometric Mean

$$\frac{a}{x} = \frac{x}{b} \quad \text{and} \quad x = \sqrt{ab}$$

Sum & Difference of Cubes

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

Binomial Theorem

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k}b^k$$

Quadratic Formula

For $ax^2 + bx + c = 0$, where $a \neq 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Parity of Sums, Differences & Products

even ± even = even	even × even = even
odd ± odd = even	odd × odd = odd
even ± odd = odd	even × odd = even

Triangular Numbers

$$T_n = \sum_{k=1}^{n} k$$

Circles

Circumference: $2\pi r = \pi d$

Area: πr^2

For radius r

Arc Length: $\frac{x}{360}(2\pi r)$

Sector Area: $\frac{x}{360}(\pi r^2)$

For central angle of x degrees

Pythagorean Triples

(3, 4, 5)

(5, 12, 13)

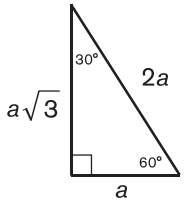
(7, 24, 25)

(8, 15, 17)

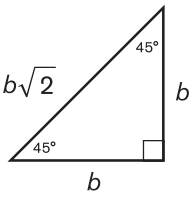
(9, 40, 41)

(12, 35, 37)

Special Right Triangles

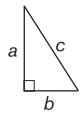


30-60-90 Right Triangle



45-45-90 Right Triangle

Pythagorean Theorem



$a^2 + b^2 = c^2$

Triangle Inequality

For a triangle with side lengths a, b and c:

$a + b > c$
$a + c > b$
$b + c > a$

Given A(x_1, y_1) and B(x_2, y_2)

Distance from A to B $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint of $\overline{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Slope of $\overline{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

Area of Polygons

Square	side length s	s^2
Rectangle	length l, width w	lw
Parallelogram	base b, height h	bh
Trapezoid	bases b_1, b_2, height h	$\frac{1}{2}(b_1 + b_2)h$
Rhombus	diagonals d_1, d_2	$\frac{1}{2}d_1d_2$
Triangle	base b, height h	$\frac{1}{2}bh$
Triangle *Heron's formula*	semiperimeter s, side lengths a, b, c	$\sqrt{s(s-a)(s-b)(s-c)}$
Equilateral Triangle	side length s	$\frac{s^2\sqrt{3}}{4}$

Polygon Angles (n sides)

Sum of the interior angle measures:

$180(n - 2)$

Central angle measure of a regular polygon:

$\frac{360}{n}$

Interior angle measure of a regular polygon:

$\frac{180(n-2)}{n}$ or $180 - \frac{360}{n}$

Solid	Dimensions	Surface Area	Volume
Cube	side length s	$6s^2$	s^3
Rectangular Prism	length l, width w, height h	$2(lw + wh + lh)$	lwh
Cylinder	circular base radius r, height h	$2\pi rh + 2\pi r^2$	$\pi r^2 h$
Cone	circular base radius r, height h	$\pi r^2 + \pi r \times \sqrt{r^2 + h^2}$	$\frac{1}{3}\pi r^2 h$
Sphere	radius r	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Pyramid	base area B, height h		$\frac{1}{3}Bh$

Equation of a Line

Standard Form

$Ax + By = C$

Slope-Intercept Form

$y = mx + b$

m = slope b = y-intercept

Point-Slope Form

$y - y_1 = m(x - x_1)$

m = slope (x_1, y_1) = point on the line