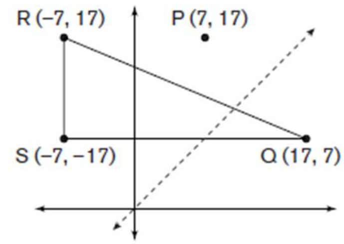
Warm-Up 9

111. There are $6^5 = 7776$ possible outcomes when rolling the die five times, and directly counting those with a sum divisible by 3 would be tedious. Instead, consider the first four rolls. There are $6^4 = 1296$ possible sequences, and their sums range from 4 to 24. If it's a multiple of 3 already, then we want the last roll of the die to be a 3 or a 6, or 1/3 of the possible rolls of the die. If the sum is 1 more than a multiple of 3 (1 mod 3), then we want the last roll of the die to be a 2 or a 5, or 1/3 of the possible rolls of the die. If the sum is 2 more than a multiple of 3 (2 mod 3), then we want the last roll of the die to be a 1 or a 4, or 1/3 of the possible rolls of the die. In each case, exactly two of the six outcomes for the last roll make the total sum divisible by 3. Thus, regardless of the first four rolls, the probability is 2/6 = **1/3** that the sum of all five rolls is divisible by 3.



112. Reflecting across the line $y = x$ causes the x and y to switch places, so the coordinates of point Q are (17, 7). Reflecting across the y-axis changes the sign of the x-coordinate, so the coordinates of point R are (−7, 17). The horizontal distance from point Q to point R is 17 − (−7) = 17 + 7 = 24 units, and the vertical distance is 17 − 7 = 10 units. By the Pythagorean theorem, $QR = \sqrt{(24^2 + 10^2)} = \sqrt{(576 + 100)} = \sqrt{676} =$ **26** units.

113. The expression $111^2 - 99^2$ is a difference of squares, so it factors as (111 + 99)(111 − 99), which then simplifies to 210 × 12 = **2520**.

114. The single-digit powers of 2 are 1, 2, 4 and 8. The only perfect square number in the 8 hundreds is $29^2 =$ **841**, and all its digits are powers of 2.

115. There are 10 possible ways to get a sum of 15 on three rolls of a standard die. The distinct permutations are 366, 636, 663, 456, 465, 546, 564, 645, 654 and 555. The probability is 2/10 = **1/5** that the first roll is a 4.

116. If the small hose takes 20 hours to fill the pool, it would fill 1/20 of the pool each hour. Likewise, if the large hose takes 16 hours to fill the pool, it would fill 1/16 of the pool each hour. Since the small hose is on for 4 hours by itself, the pool was already 4/20 full at 12:00 p.m. when the large hose was turned on. Together, the two hoses will fill 1/20 + 1/16 = 4/80 + 5/80 = 9/80 of the pool each hour, or 4 × 9/80 = 9/20 of the pool in 4 hours. By 4:00 p.m., the pool must be 4/20 + 9/20 = 13/20 or **65**% full.

117. The probability that person A tells the truth and person B lies is 3/4 × 1/5 = 3/20, and the probability that person A lies and person B tells the truth is 1/4 × 4/5 = 4/20. Therefore, the probability that one tells the truth and the other lies is 3/20 + 4/20 = **7/20**.

118. Let's call the first term of this sequence a and the second term b. The third term is then $a + b$, and the fourth term is ab. Since this is an arithmetic sequence, there must be a common difference between terms. Subtracting the second term from the third term, we get $(a + b) - b = a$, which shows that this common difference is equal to the first term, a. The difference between the second term and the first term must also be a, so we get $b - a = a$, which means that $b = 2a$. Now we can look at the difference between the fourth and the third terms, which is $ab - (a + b) = a$, and substitute $2a$ for b. This gives us the equation $2a^2 - (a + 2a) = a$, which we can simplify to $2a^2 - 4a = 0$, then $a^2 - 2a = 0$. We can factor this as $a(a - 2) = 0$, so $a = 0$ and $a = 2$. Since we know that the sequence consists "entirely of positive integers," we can discard the $a = 0$ solution and keep the $a = 2$ solution. The first four terms of the sequence are 2, 4, 6, 8, and the fifth term is **10**.

119. The area of rectangle ABCD is 5 × 3 = 15 cm^2. The area of triangle EBF is 1/2 × 4 × 2 = 4 cm^2. The area of pentagon AEFCD is the difference 15 − 4 = **11** cm^2.

120. The value of $13^2 - 12^2 - 5^2$ is 169 − 144 − 25 = **0**.