Pi Problems

1. _____ March 14 (3/14) is often celebrated as Pi Day since π is estimated to be about 3.14. However, in reality, when π is written as a decimal, it is a number that never ends and never repeats. Therefore, many people enjoy memorizing as many digits of π as they can. The 2003 School Competition had a question related to this: In March 2000, a high school senior in Fargo, N.D. won his school's annual Pi Memorization Contest by reciting 5005 digits of π. He recited the digits in 55 minutes. On average, how many digits did he recite each minute?

2. _____ Since we don't all have time to memorize that many digits of π, it is often helpful to use approximations. Around 2000 BC the Babylonians approximated the value of π to be $3\frac{1}{8}$, and the Egyptians approximated the value of π to be $4(\frac{8}{9})^2$. What is the positive difference between these two approximations of the value of π? Express your answer as a decimal to the nearest thousandth.

3. _____ The value of π comes from the ratio of the circumference of a circle to its diameter. Mary decided to approximate the value of π by collecting some data. The data that follow are her measurements for the circumference and diameter, in mm, of five different circles. What is the mean of the circumference-to-diameter ratios of these circles? Express your answer as a decimal to the nearest thousandth.

Circle	I	II	III	IV	V
Circumference, mm (C)	89	67	58	35	97
Diameter, mm (D)	29	21	18	11	31
Circum. to Diam. (C/D)					

For #4 and #5, students should use the π button on their calculator rather than any approximation for π.

4. _____ Cory's buggy has front wheels each with a diameter of 12 inches and back wheels each with a diameter of 18 inches. If Cory pushes the buggy for 300 yards, how many more revolutions than each of the back wheels does each of the front wheels make? Express your answer as a decimal to the nearest tenth.

5. _____ Semicircle A's radius is twice as long as semicircle B's radius. The length of semicircle B's radius is 20% of the length of semicircle C's radius. None of the semicircles overlap one another. If semicircle A's radius is 10 cm, what is the sum of the areas of semicircles A, B and C, in sq cm? Express your answer as a decimal to the nearest tenth.

St. Patrick's Day Problems
March 17 is St. Patrick's Day. We wish you the luck o' the Irish as you tackle these problems.

6. _____ How many four-leaf clovers would need to be placed on the right side of the third scale to balance it?
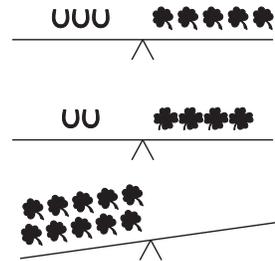
7. _____ Riley sees a rainbow that reaches a maximum height of 0.5 miles above the ground and has ends that appear to touch the ground 1 mile apart. If the rainbow is an arc of a circle, how many degrees is the arc that Riley sees from the point it leaves the ground to the point it returns to the ground?

8. _____ Patti writes "Saint Patrick's Day" on a strip of paper and cuts it so that each of the 16 letters is on its own piece of paper. (She discards the apostrophe.) If she puts all of the letters in a hat and will draw out exactly five letters at random and without replacement, what is the probability that she will draw all five letters of her name? Express your answer as a common fraction.

9. _____ Margaret painted a mural for St. Patrick's Day and mixed her own green paint. She used a ratio of 3 parts yellow to 2 parts blue. Maureen also wanted to paint a mural with the same green color, but she currently has 8 cups of green paint that is a mixture of 40% yellow paint and 60% blue paint. To get the same shade of green as Margaret, how many cups of yellow paint must she add to her mixture?

10. _____ Let's now take a look at the word GREEN. There are not too many real words that can be made from rearranging the letters in GREEN. For this problem, though, let's see how many distinct ways we can arrange the five letters in the word GREEN, even if they don't form real words. Let's also add a restriction: Any arrangement must keep the two Es together. How many such arrangements exist?

St. "Pi"Tricks Day: Solutions

Problem 1. We are looking for the ratio of the number of digits per minute. Therefore, our answer can by calculated by changing the ratio 5005/55 to a ratio with a denominator equal to 1. These numbers actually work out very nicely, and by dividing both the numerator and denominator by 55, we see that he averaged 91 digits per minute.

Problem 2. 3 1/8 = 3.125, 4(8/9)^2 = 3.160, and 3.160 – 3.125 = 0.035

Problem 3.

| Circle	A	B	C	D	E
Circumference mm	89	67	58	35	97
Diameter mm	29	21	18	11	31
Circumference ÷ Diameter	*3.069*	*3.191*	*3.222*	*3.182*	*3.129*

(3.069 + 3.191 + 3.222 + 3.182 + 3.129) ÷ 5 = 3.1586. The mean of the circumference to diameter ratios is 3.159 to the nearest thousandth.

Problem 4. First find the circumference of each of the wheels. C = πd. C = (3.14)(12) = 37.68 inches. C = (3.14)(18) = 56.52 inches. Now convert the number of yards traveled by the buggy to inches. 300 yds (3 ft/yd) (12 in/ft) = 10800 inches. Divide the distance travelled (in inches) by the distance travelled in one revolution of each wheel.10800/37.68 = 286.624, 10800/56.52 = 191.083. Finally, find the difference. 286.624 - 191.083 = 95.5 revolutions, to the nearest tenth.

Problem 5. If semicircle A's radius is 10 cm, we know that B's radius is 5cm and that C's radius is 16.66667cm. Now we can find the area of each semicircle.A = (πr^2)/2. A = (3.14)(10)^2/2 = 157. A = (3.14)(5)^2/2 = 39.25. A = (3.14)(16.66667)^2/2 = 3935/9 or 436.1111… Now find the sum. (3935/9) + (157) + (39.25) = 22765/36 or 632.36111… NOTE: your final answer may differ SLIGHTLY because of differences in rounding on this problem.

Problem 6. Since we know we have 10 three-leafed clovers we can see that's equivalent to 6 horse shoes. By looking at the next scale we see that 6 horse shoes are equivalent to 15 four-leafed clovers. So 10 three-leafed clovers equal 15 four-leafed clovers.

Problem 7. First let's draw a diagram.
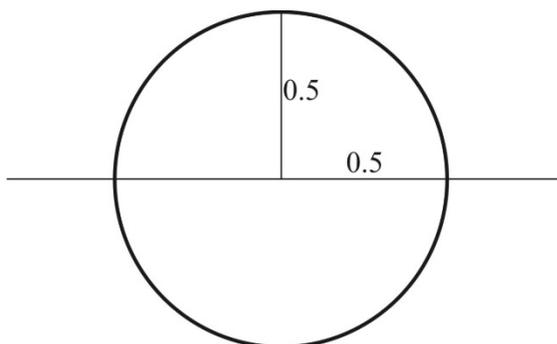
We quickly see that the only way for the chord length to be twice the length of the highest distance from the center of the circle is if the chord is also the diameter. This means that the arc measure must be 180°.

Problem 8. When finding probability you are essentially finding the "number of desired outcome possibilities"/"the total number of outcome possibilities". For this question we need to know how many arrangements of the letters of Patti's name exist and the total possible number of 5-letter arrangements that exist using the letters from the words Saint Patricks Day. Using the letters in Patti's name there are $(5!)(3)(2) = 720$ possible arrangements. (Note: we multiply by 3 because there are 3 a's available in "Saint Patricks Day," we multiply by 2 because there are 2 i's available in "Saint Patricks Day"). The total number of arrangements is $(16)(15)(14)(13)(12) = 524,160$. Therefore the probability of drawing the letters of her name is $720/524,160 = 1/728$

Problem 9. Maureen currently has $(0.40)(8) = 3.2$ cups of yellow paint and $(0.60)(8) = 4.8$ cups of blue paint in her mixture. We know that she needs more yellow paint, so she will eventually have $3.2 + x$ cups of yellow paint and 4.8 cups of blue paint, and we want this to be a ratio of 3:2. So we can set up the proportion $(3.2 + x) / 4.8 = 3 / 2$. If we find the cross-products, we get $6.4 + 2x = 14.4$. Subtracting 6.4 from both sides leads to $2x = 8$ and $x = 4$. Therefore, Maureen needs to add 4 cups of yellow paint to the current mixture to get Margaret's shade of green.

Problem 10. If we're keeping the two Es together, then we might as well just replace EE with, let's say, a P. So now we just need to figure out how many arrangements of the letters G, R P and N exist. There are four letters, so there are $4! = 24$ ways they can be arranged. A tree diagram may be helpful to visualize this, but make sure it's big enough for 24 branches!