Warm-Up 8

101. Brennan will need 4 pennies to make 1, 2, 3 and 4 cents. If he also has 1 nickel, he can make 5 through 9 cents. Adding a dime allows him to make amounts from 10 through 19 cents. With his last coin being another dime, Brennan can make any amount from 20 to 25 cents. So, with just **7** coins—4 pennies, 1 nickel and 2 dimes—Brennan can pay any amount from 1 to 25 cents in exact change.

102. The area of the inner circle is πr^2 and the area of the outer circle is $\pi(2r)^2 = 4\pi r^2$. The area of the annulus (or ring) is $4\pi r^2 - \pi r^2 = 3\pi r^2$. We need some fraction of $3\pi r^2$ to be equal to 1/8 of $4\pi r^2$. That fraction should have a denominator of 360 so that the numerator is the number of degrees in the unknown angle x. We need to solve the following equation for x: $x/360 \times 3\pi r^2 = 1/8 \times 4\pi r^2$. This simplifies to $x/120 = 1/2$, so $x =$ **60** degrees.

103. There are $360 \times 60 \times 60$ arcseconds in a full circle and $24 \times 60 \times 60$ seconds in 24 hours. The circle will rotate $360 \div 24 =$ **15** arcseconds every second.

104. The triangle inequality theorem states that the sum of the lengths of any two sides of a triangle must be greater than the length of the third side. The list to the right shows the only **5** non-congruent triangles with integer side lengths and a perimeter of at most 7.

(1, 1, 1)
(1, 2, 2)
(1, 3, 3)
(2, 2, 2)
(2, 2, 3)

105. A square with a side length of s has a perimeter of $4s$ and an area of s^2. We are told that the perimeter exceeds the area by a positive integer n, so we have the equation $4s = s^2 + n$. Rearranging gives $s^2 - 4s = -n$, and completing the square yields $(s - 2)^2 = 4 - n$. Since the square of a real number is nonnegative, we must have $4 - n \geq 0$, so $n \leq 4$. Since n is a positive integer, the possible values of n are 1, 2, 3 and 4. Each of these values gives a real solution for s, so all are valid. Therefore, the sum of all possible distinct values of n is $1 + 2 + 3 + 4 =$ **10**.

106. We can take the cross product of the given equation to get $(x - 1)(x - 5) = (x - 3)(x - 4)$. We can now expand both sides of the equation to get $x^2 - 6x + 5 = x^2 - 7x + 12$. We can subtract x^2 from both sides of the equation to get $-6x + 5 = -7x + 12$, and finally we can add $7x - 5$ to both sides to get $x =$ **7**. Note that substituting $x = 7$ into the original equation does not make any denominator equal to zero, so it is not an extraneous solution.

107. If the frog is to get across the river in the fewest hops, he should take as many triple hops as possible. With 18 lily pads in the pond, it will take 19 hops to get across. The frog can do 5 triple hops and 2 double hops, since $5 \times 3 + 2 \times 2 = 19$. The sequence TTTTTDD describes one way the frog could get across the pond and TTTTDTD describes another. We want to count all the ways that 2 of these 7 characters can be D instead of T. This is just "7 choose 2," which is $7 \times 6 \div 2 = 21$. Therefore, there are **21** different ways the frog can hop from one shore to the other.

108. The ratio of flour to eggs in Rachel's pumpkin bread recipe is 3.5/4 or 7/8, which means there are 7/8 as many cups of flour as there are eggs in the recipe. If Rachel uses 7 eggs, she should use $7/8 \times 7 = 49/8 =$ **6 1/8** cups of flour.

109. The positive integer divisors of 32 are 1, 2, 4, 8, 16 and 32, of which three are perfect squares: 1, 4 and 16. If one of the divisors is chosen at random, the probability is 3/6 or **1/2** that the chosen divisor is a perfect square.

110. If Leona had 85 cents entirely in nickels, then she would have $85 \div 5 = 17$ coins. Each time she trades 2 nickels for 1 dime, she reduces the total number of coins by 1 because 2 coins become 1 coin. Since she ends up with 12 coins, the total number of trades must be $17 - 12 = 5$. That means she traded 10 nickels (5 trades × 2 nickels each) for 5 dimes. Therefore, she must have **5** dimes.