U.C. MATH BOWL 2023

LEVEL III — Session 1

Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. You ordered a family size pizza and 40 minutes later, the delivery arrived ... but with a small size pizza instead! You called the shop and the owner said, "SO sorry! I'll send you another small size pizza." Is his offer fair to you? Explain why or why not.

Size	smalll	medium	large	family
inches (diameter)	12	14	16	18

Pizzas are flat so it is enough to consider the area. Is $2 \times 12^2 \geq 18^2$?

2. There are three square pieces of bread, all of the same size, and a knife. What is the minimum number of cuts you need to make to divide them fairly among four people?

   You are allowed fold each piece of bread and put them on top of one another if you want.

Stack them and make 2 cuts on the diagonals. That gives you 12 congruent pieces; give everyone 3. That lets you do it with two cuts and is enough to get most of the points on this problem.

A better solution is to stack the squares and cut the stack parallel to a side and 1/4 of the way along the other side thus producing 3 pieces which are 3/4 of the square and 3 pieces which are each 1/4 of a square. Give three people the big pieces and the fourth person the 3 tiny pieces. That way everyone gets 3/4 of a piece of bread and you only need one cut.

3. If you jog 1 mile at 4 miles per hour and jog another mile at 8 miles per hour and then run the last two miles at 12 miles per hour, what is your average speed for the four miles?

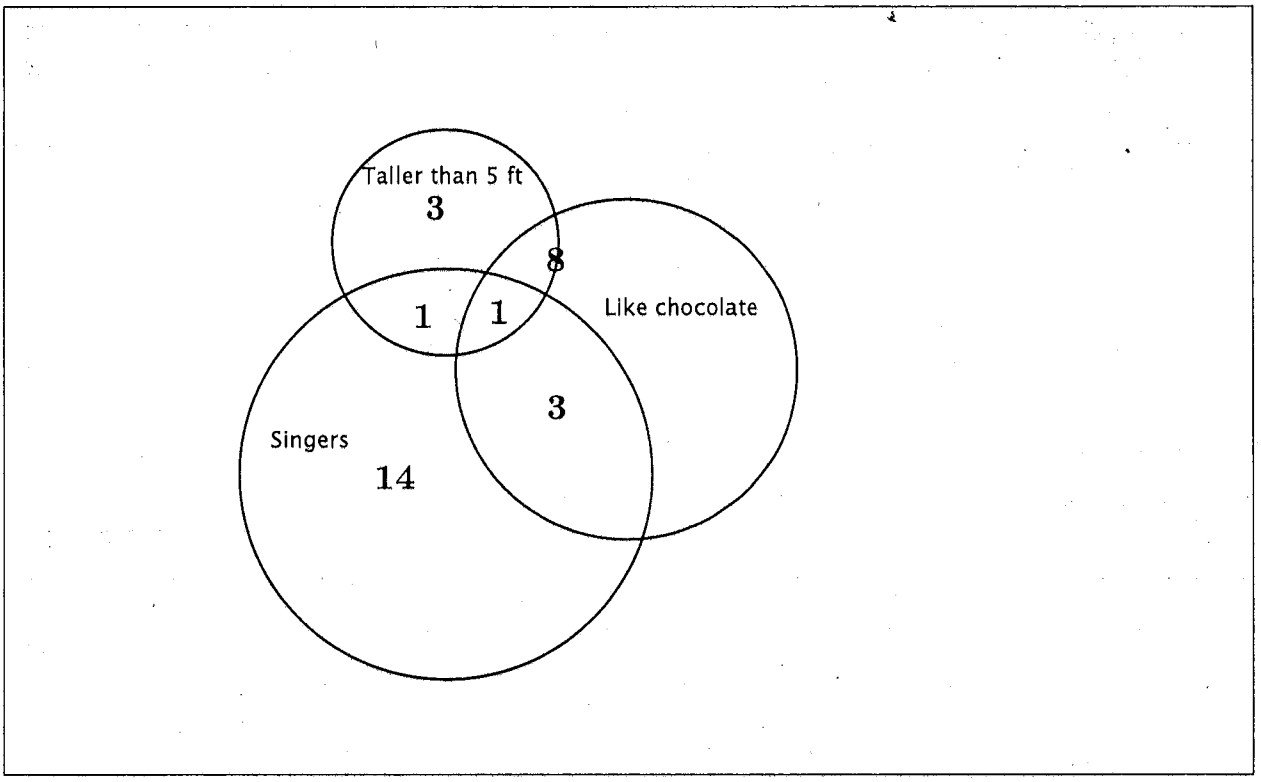
The first mile took 1/4 hour The second took 1/8 hour and the last two miles took 1/6 of an hour. In all you went 4 miles in $1/4 + 1/8 + 1/6$ hours for an average speed of

$$\frac{4}{1/4 + 1/8 + 1/6}$$

miles per hour.

4. In a 3rd grade class with 30 students every student is described by at least one of these phrases (1) Taller than 5 feet, (2) Likes chocolate, (3) Great singer. 12 student like chocolate, 4 of whom are great singers. There are 2 great singers that are over 5 feet but only one of

them likes chocolate. There are 14 singers shorter than 5 feet who do not like chocolate. How many students are there who are over 5 feet, don't like chocolate, and are not great singers?



5. A *palindrome* is a whole number that reads the same forwards as backwards (for example 121 and 1441 and 3333 are palindromes). If we neglect the colon, certain times displayed on a digital clock are palindromes. Three examples are 1:01, 4:44, 12:21. How many times during a 12 hour period starting at midnight will there be palindromes shown on the face of a digital clock?

If your digital clock is the kind that counts hours starting at 0 : 00 for midnight you'll get one answer (62) to this question; if your clock starts with midnight as 12 : 00 you'll get a different answer (57). In either case for each hour 1 through 9 there are 6 palindromes: 1:01,1:11,1:21,...1:51,2:02,...2:52,...9:09,..., 9:59; That makes 54 palindromes.

If midnight is 12:00 the remaining palindromes are 12:21,10:01, and 11:11. This gives $54 + 3 = 57$ palindromes.

If midnight is 0:00 there are 6 palindromes in hour 0:00 and then 10:01, and 11:11. This gives $54 + 8 = 62$ palindromes.

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LEVEL III — Session 2

Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. When each side of a square is increased by 50% the area is increased by 180 square inches. What was the area of the original square?

If the side of the original square was x we're told that $(1.5x)^2 - x^2 = 180$. That says $(5/4)x^2 = 180$ so $x^2 = 720/5 = 144$. So $x = 12$ and the original square had area 144 square inches.

2. Decode the following cipher of an English sentence, where each alphabet letter has been replaced by some other alphabet letter.

COXBX TBX CVK CDGXR DI T GTI'R ADHX VOXI OX ROKQAU IKC RNX-PQATCX: VOXI OX PTI'C THHKBU DC, TIU VOXI OX PTI.

THERE ARE TWO TIMES IN A MAN'S LIFE WHEN HE SHOULD NOT SPECULATE: WHEN HE CAN'T AFFORD IT, AND WHEN HE CAN.

Guesses based on the frequency with which letters appear in the cipher (i.e. the most frequent has probably been substituted for 'e' ...) will get you there. Noticing what comes after apostrophes (Is that a contraction (t) or a possessive (s)?) also help a bit.

3. If Michael tosses a die twice (i.e., one toss will give a number), he will get two integers in the range $1 \ldots 6$. Find the probability that the result of multiplying these two numbers is a multiple of 3.

Answer is $\frac{5}{9}$.

To see this we could enumerate all events that have a result being multiple of 3:

$$(1,3),(2,3),(3,3),(4,3),(5,3),(6,3),\cdots,(3,4),(3,5),(6,1),(6,2),(6,4),(6,5)$$

finding a total of 20. Since there are a total of 36 possible results the probability is $\frac{20}{36}$.

Alternatively, we could think that the probability that the first roll has a factor of 3 which is is $1/3$, and that the second roll has a factor of 3 is also $1/3$ while the chance that they both do is $1/9$ giving an overall probability of $1/3 + 1/3 - 1/9 = 5/9$.

4. What is the smallest number of coins (only pennies, nickels, dimes, and quarters are allowed) needed so that you can make up every amount from 1 cent to 1 dollar?

Ten coins. You need 4 pennies to make up numbers 1-4. Add a nickel and you can now make up 1-9. Add a dime, and you can make up 1-19. Add another dime (too early to add a quarter) and you make up 1-29. Add a quarter, and you can now make up 1-54 (check!). Another quarter makes is 1-79 (check!). You now need to at least one more coin and adding a quarter gets you every number 1 — 100.

5. In a certain country every 20th mathematician is a writer while every 45th writer is a mathematician. Are there more writers or mathematicians in this country? How many times more?

If m, w are the numbers of mathematicians and writers and b is the number that are both, we're told that $b = m/20$ and $b = w/45$ so

$$m/20 = w/45$$

showing that $w = 9m/4$. There are $9/4$ as many writers as mathematicians.