MATHCOUNTS®

2020
■ State Competition ■
Team Round
Problems 1–10

School _____

Chapter _____

Team Members _____ , Captain

_____

_____

_____

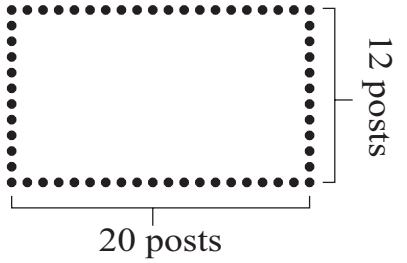
DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk to each other during this section of the competition. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers on his/her own competition booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

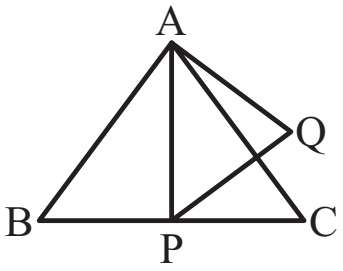
1. _____ What is the smallest five-digit positive integer whose digits are all even and have a sum of 14?

2. _____ meters Melinda and her father are building a rectangular horse pen that will have 12 posts along two sides and 20 posts along the other two sides, as shown. The posts will be 12 meters apart with three wires: one wire near the top, one in the middle and one near the bottom. Besides the wire between the posts, an additional half meter of wire is needed to wrap around each post on each level. How many meters of wire will be required to make the pen?



3. _____ % In basketball, a field goal can be worth either two or three points. During his career, an NBA player has made 49.93% of his two-point field goal attempts and 36.51% of his three-point field goal attempts. Assuming the value of a missed attempt is 0, the average value of this player's career field goal attempts is 1.0389 points. What percent of his career field goal attempts have been three-point attempts? Express your answer as a percent to the nearest whole number.

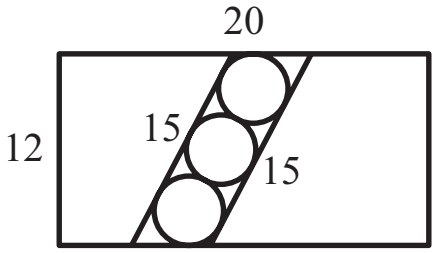
4. _____ degrees Coplanar isosceles triangles ABC and QAP are similar with vertex angles A and Q, respectively. If segment AP is perpendicular to segment BC and the $m\angle QAC = 16°$, what is $m\angle PAQ$, in degrees?



5. _____ The operation ☆ is defined on ordered pairs as $\bigstar(a, b) = (a + 3b, 9a - 5b)$. If x and y are nonzero real numbers such that $\bigstar(x, y) = (-8x, -8y)$, what is the value of the ratio $x:y$? Express your answer as a common fraction.

6. (,) Line segment AB, with endpoints A(1, 1), B (10, 8), is rotated about point C(x, y) to its image A′B′ with endpoints A′(1, 11), B′(12, 8). What are the coordinates of point C? Express your answer as an ordered pair.

7. _____ feet A parallelogram with three congruent inscribed circles is painted across a 20-foot by 12-foot rectangular banner as shown. The longer sides of the parallelogram have length 15 feet. What is the radius of each circle? Express your answer as a common fraction.



8. _____ integers The *prime-sum radical* of a number n is the sum of the distinct prime divisors of n. For example, the prime-sum radical of 45 is 8 and the prime-sum radical of 42 is 12. How many integers from 2 to 1000, inclusive, have a prime-sum radical of 10?

9. _____ ways A cheerleading squad has 14 cheerleaders, each a different height. How many ways are there for the cheerleaders to line up for a photo in two rows with seven people each, so that each cheerleader in the back row is taller than the one immediately in front of them, and so that the heights of the cheerleaders in the back row descend from the middle to each side?

10. _____ % If six numbers are chosen at random, with replacement, from the set of integers from 1 to 900, inclusive, what is the probability that the product of these six integers leaves a remainder of 4 when divided by 30? Express your answer as a percent to the nearest thousandth.