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# MATHCOUNTS®

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2020  
■ State Competition ■  
Target Round  
Problems 1 & 2

Name \_\_\_\_\_

School \_\_\_\_\_

Chapter \_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.**

This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the problem sheets. If you complete the problems before time is called, use the time remaining to check your answers.

Problem 1	Problem 2	Scorer's Initials

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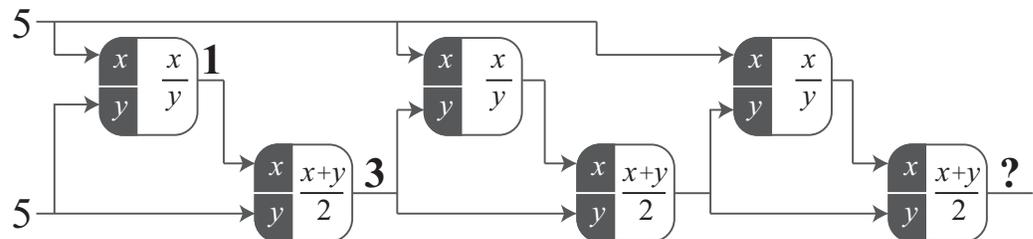
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1. \_\_\_\_\_ days

The cheapest available ticket for Super Bowl LI on the secondary market costs \$2400. Suppose Joel earns the federal minimum wage of \$7.25 per hour, 20% of which is deducted for taxes, and he works from 9 a.m. to 5 p.m. each day. How many days must Joel work to pay for a single ticket? Express your answer to the nearest whole number.

2. \_\_\_\_\_

Six function machines are connected as shown. Three of the function machines are “dividers”, while three of the function machines are “averagers”. Each divider takes two inputs (into the left) and sends their quotient to the output (on the right). Each averager takes two inputs (into the left) and sends their arithmetic mean to the output (on the right). Given that the initial inputs are  $x = 5$  and  $y = 5$ , as shown, what is the final result emitted by the right-most function machine? Express your answer as a mixed number.



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# MATHCOUNTS®

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2020  
■ State Competition ■  
Target Round  
Problems 3 & 4

Name \_\_\_\_\_

School \_\_\_\_\_

Chapter \_\_\_\_\_

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Problem 3	Problem 4	Scorer's Initials

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3. \_\_\_\_\_ There exist infinitely many common fractions  $\frac{a}{b}$ , where  $a > 0$  and  $b > 0$  and for which  $\frac{3}{5} < \frac{a}{b} < \frac{2}{3}$ . Of these common fractions, which has the smallest denominator? Express your answer as a common fraction.

4. \_\_\_\_\_ A sequence is defined by  $S_n = 1 + 2 + 3 + \dots + (n - 1) + n$  for all positive integers  $n$ . Another sequence is defined as  $P_n = 2S_n \times 2S_{n+1} \times 2S_{n+2} \times \dots \times 2S_{2n-1}$  for all positive integers  $n$ . The value of  $\sqrt{P_{2019}}$  can be written in simplest radical form as  $a\sqrt{b}$ , where  $a$  and  $b$  are positive integers. What is the value of  $b$ ?

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# MATHCOUNTS®

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2020  
■ State Competition ■  
Target Round  
Problems 5 & 6

Name \_\_\_\_\_

School \_\_\_\_\_

Chapter \_\_\_\_\_

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Problem 5	Problem 6	Scorer's Initials

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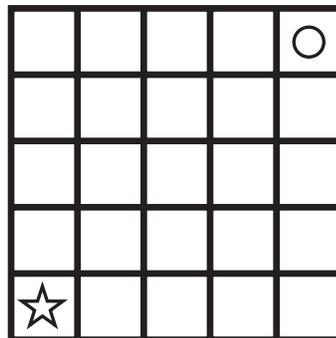
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5. liars A remote island is populated by both truth-tellers and liars. Truth-tellers always tell the truth, and liars always lie. To conduct a census of the island, Erin gathers all 72 of its inhabitants and asks each person how many truth-tellers and liars there are. She receives 61 responses saying “there are 32 truth-tellers and 40 liars” and 11 responses saying “there are 11 truth-tellers and 61 liars”. How many liars are there?

6. sequences Iris is playing a game that has a  $5 \times 5$  gameboard like the one shown. The goal is to get her game piece from the square labeled  $\star$  to the square labeled  $\circ$  using a series of moves any positive integer number of squares up or any positive integer number of squares to the right. Note that moving two squares up in a single move is different than moving two squares up in two moves. How many unique sequences of moves can Iris make to get her game piece from the  $\star$  to the  $\circ$ ?



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# MATHCOUNTS®

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2020  
■ State Competition ■  
Target Round  
Problems 7 & 8

Name \_\_\_\_\_

School \_\_\_\_\_

Chapter \_\_\_\_\_

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Problem 7	Problem 8	Scorer's Initials

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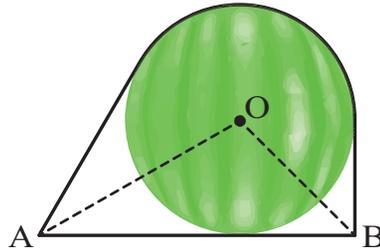
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7. \_\_\_\_\_ cm    The figure shows a perfectly round watermelon, represented by circle O, secured on a flat table, represented by segment AB, using a rope. One end of the rope is attached to the table at A, then draped over the top of the watermelon, and the other end is attached at B. The watermelon has diameter 20 cm. The center of the watermelon is 20 cm from point A and  $10\sqrt{2}$  cm from point B. What is the minimum length of rope necessary to secure the watermelon to the table? Express your answer as a decimal to the nearest tenth.



8. \_\_\_\_\_ times    Tyrell makes a list of the left-most digit of the powers of 2, from  $2^0$  to  $2^{1000}$ , inclusive. The first six numbers on Tyrell's list, therefore, are 1, 2, 4, 8, 1, 3 and the last six numbers are 3, 6, 1, 2, 5, 1. Given that  $2^{1000}$  has 302 digits, and that 8 appears 52 times on Tyrell's list, how many times does 9 appear in his list?