# MATHCOUNTS®

## 2020
## ■ State Competition ■
## Sprint Round
## Problems 1–30

**HONOR PLEDGE**

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

School _____

Chapter _____

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.**

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

| Total Correct | Scorer's Initials |
|---|---|
| | |
| | |

1. _____ degrees

In triangle ABC, $m\angle A = 60°$, $m\angle B = 100°$ and $m\angle C = 20°$. If segment BD is constructed inside this triangle so that it bisects $\angle ABC$, what is the degree measure of $\angle BDC$?

2. _____

What is the value of the expression $\dfrac{1.2\times10^2}{4.8\times10^5}$? Express your answer as a common fraction.

3. _____

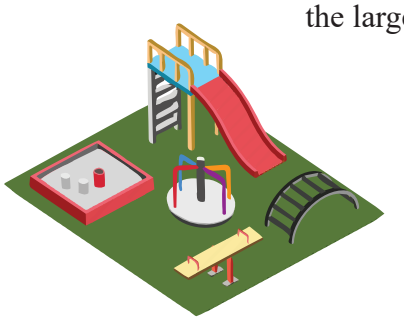
If ⚾ + ✈ = 14, and ⚾ − ✈ = 4, what is the value of ✈?

4. _____ units

Hexagon ABCDEF is drawn in the coordinate plane with vertices A(0, 0), B(4, 0), C(4, 2), D(2, 2), E(2, 4) and F(0, 4). What is the perimeter of hexagon ABCDEF?

5. _____ square tiles

A school wishes to use square tiles of artificial turf to cover an outdoor play area that measures 40 feet by 72 feet. Only whole tiles that are congruent squares of the largest possible size will be used. How many such square tiles are needed?

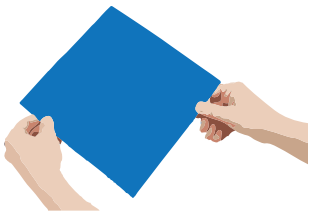


6. _____ miles

If Rose and Robyn ran 29 miles combined, and Robyn ran 4 miles less than twice as much as Rose, how many miles did Rose run?

7. _____ inches

Jamie folds a piece of paper in half, and then folds it in half again. If the resulting folded piece of paper is a 4-inch by 5-inch rectangle, what is the largest possible perimeter of Jamie's original unfolded piece of paper?



8. _____

If $k! = 2^7 \cdot 3^2 \cdot 5 \cdot 7$, what is the value of $k$?

9. _____

The square of 207 can be computed by multiplying 200 by $200 + x$ and adding 49. What is the value of $x$?

10. _____ units

A, B, C, D, E, F and G are points on the same line and AB = 24 units. The midpoint of segment AB is C. The midpoint of segment CB is E. The midpoint of segment AE is D. The midpoint of segment AC is F. The midpoint of segment EB is G. What is the length of segment FG?

11. _____ minutes

Starting at 9:00 p.m., Joe watches three films consecutively, with no breaks in between. The first film is the same length as the third film, and the second film is 50% longer than each of the other two films. If Joe finishes the third film at 2:15 a.m., how many minutes long is the second film?

12. _____ %

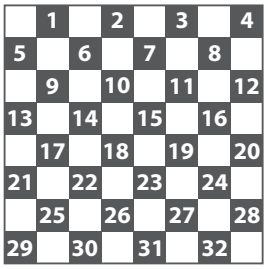
If $d$ is 10% greater than $c$, $c$ is 25% less than $b$, and $a$ is 50% greater than $b$, by what percent is $d$ less than $a$?

13. _____

If $A$, $B$, $C$, $D$, $E$, $F$ and $G$ satisfy the equations shown, what is the value of the absolute difference between $A + G$ and $A - G$?

$$\begin{aligned} A+B+C &= 5 \\ B+C+D &= 7 \\ C+D+E &= 9 \\ D+E+F &= 11 \\ E+F+G &= 13 \\ F+G &= 10 \\ A+F+G &= 11 \end{aligned}$$

14. _____ ways



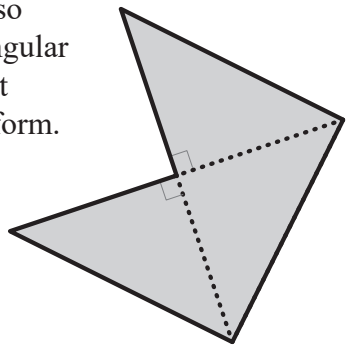

A checker starts at square 4 of the checkerboard shown here. At any time, it can move to any diagonally adjacent square below its current position. How many possible ways are there for the checker to move from square 4 to square 32?

15. _____ units$^2$

Sanjay cuts off a corner from a rectangular piece of paper, forming a pentagon. The side lengths of this pentagon, in order, are 10 units, 2 units, 11 units, 10 units and 5 units. What is the area of this pentagon?

16. _____ The sum of nine nonnegative numbers is 200. If $M$ and $A$ represent the median and arithmetic mean of the nine numbers, respectively, what is the greatest possible value of $M - A$? Express your answer as a common fraction.

17. _____ $\text{in}^2$ A five-sided piece of paper is divided into three congruent right triangles as shown in this figure, in which the dotted segments have length 4 inches. If the piece of paper is folded along the dotted segments so that the triangles become three faces of a right triangular pyramid, what is the area of the missing face of that pyramid? Express your answer in simplest radical form.



18. _____ If the equation shown is true for every value of $x$, what is the value of $b + c$?

$$(x^3 + x^2 - 3x + b)(2x^4 + cx^3 + x^2 - x + 1) = 2x^7 + 8x^6 + x^5 - 4x^4 + 39x^3 + 11x^2 - 10x + 7$$

19. _____ Suppose $a_0 = 10$ and $a_{n+1} = a_n^2$ for every non-negative integer $n$. What is the smallest value of $n$ for which $a_1 \cdot a_2 \cdot a_3 \cdots a_{n-1} \cdot a_n$ has at least 100 digits?

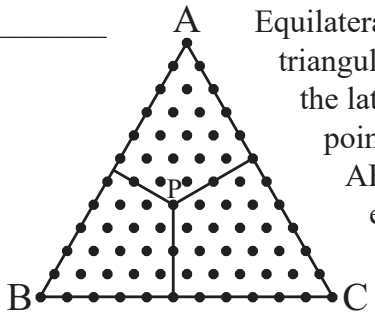
20. _____ positive integers How many of the first 1024 positive integers are neither perfect squares nor perfect fifth-powers?

21. _____ Call a multi-digit positive integer *divisorly* if, for each pair of adjacent digits A and B, either A = B$n$ or B = A$n$ for some integer $n$. For example, 12639 is divisorly because 1 evenly divides 2, 2 evenly divides 6, 3 evenly divides 6, and 3 evenly divides 9. What is the greatest divisorly integer whose digits are all different?

22. _____ The lines $x + y = a$, $3x + 2y = a + 15$ and $4x + 5y = a + 19$, when graphed in the $xy$-plane, all intersect at the same point. What is the value of $a$? Express your answer as a common fraction.

23. _____ All of the permutations of the digits 1, 3, 5, 7 and 9 are listed in numerical order from 13,579 to 97,531. What is the 100th permutation in the list?

24. _____ Erica chooses seven not necessarily distinct digits A, B, C, D, E, F and G, and forms the seven 4-digit numbers: ABCD, BCDE, CDEF, DEFG, EFGA, FGAB, GABC. If $S$ is the sum of Erica's seven numbers, what is the greatest possible prime factor of $S$?

25. _____ 

Equilateral triangle ABC with side length 11 is constructed on a triangular lattice grid with 12 lattice points are on each side. One of the lattice points inside the triangle is randomly selected and labeled point P. If $a$, $b$ and $c$ are the distances from P to sides BC, AC and AB, respectively, as shown, what is the probability that a triangle exists with side lengths $a$, $b$ and $c$? Express your answer as a common fraction.

26. _____ A circle is tangent to the positive $x$-axis at $x = 3$. It passes through the distinct points (6, 6) and ($p$, $p$). What is the value of $p$? Express your answer as a common fraction.

27. _____ positive integers How many of the first 2019 positive integers have no odd single-digit prime factors?

28. _____ What is the value of $\sqrt{11{,}111{,}111 \times 100{,}000{,}011 + 4}$ ?

29. _____ David throws a dart at a triangular dartboard whose side lengths are 5, 5 and 6, and the dart lands in a random location on the dartboard. What is the probability that the sum of the squares of the 3 distances from the dart's location to the corners of the dartboard is less than 30? Express your answer as a common fraction in terms of $\pi$.

30. _____ Hank builds an increasing sequence of positive integers as follows: The first term is 1 and the second term is 2. Each subsequent term is the smallest positive integer that does NOT form a three-term arithmetic sequence with any previous terms of the sequence. The first five terms of Hank's sequence are 1, 2, 4, 5, 10. How many of the first 729 positive integers are terms in Hank's sequence?