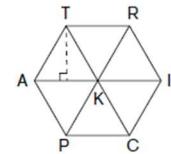


## Workout 3

161. Suppose the lengths of the sides of the rectangle are  $a$  and  $b$ . Then  $ab = 20$  and  $a + b = 26/2 = 13$ . The new rectangle will have area  $(a + 1)(b + 1) = ab + a + b + 1$ . So, the area of the rectangle is  $20 + 13 + 1 = \mathbf{34} \text{ cm}^2$ .

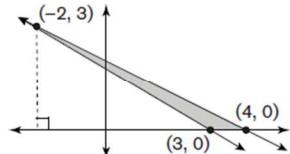
162. The perimeter of heptagon PATRICK is 28 inches, so each segment must be  $28 \div 7 = 4$  inches. The area of the heptagon consists of 5 equilateral triangles. Each equilateral triangle has a base of 4 inches, a height of  $2\sqrt{3}$  inches, and an area of  $1/2 \times 4 \times 2\sqrt{3} = 4\sqrt{3}$  square inches. Thus, the area of heptagon PATRICK is  $5 \times 4\sqrt{3} = \mathbf{20\sqrt{3} \text{ in}^2}$ .



163. There are 8 letters in the word TOPOLOGY, but 3 letters are the same, so there are  $8! \div 3! = 40,320 \div 6 = \mathbf{6720}$  ways to rearrange the letters.

164. At one extreme, the set could contain the number 27 by itself, so  $d = 1$  is a possibility. At the other extreme, the set could contain 2, 3, 4, 5, 6 and 7, whose sum is 27, so  $d = 6$  is also a possibility. With some checking, we can confirm that  $d$  can also be any natural number between 1 and 6. The sum of the possible values of  $d$  is thus  $1 + 2 + 3 + 4 + 5 + 6 = \mathbf{21}$ .

165. The line  $x + 2y = 4$  intersects the  $x$ -axis at  $(4, 0)$ , the line  $3x + 5y = 9$  intersects the  $x$ -axis at  $(3, 0)$ , and their point of intersection is  $(-2, 3)$ . The triangle formed by these three points has a base on the  $x$ -axis of length 1 unit and a height of 3 units. Thus, the area of this triangle is  $(1 \times 3) \div 2 = \mathbf{3/2}$  units $^2$ .



166. The prime factorization of 2026 is  $2 \times 1013$ , so the only possible dimensions for George's prism are 1 cm by 2 cm by 1013 cm. Therefore, the surface area is  $2(1 \times 2 + 1 \times 1013 + 2 \times 1013) = 2(2 + 1013 + 2026) = 2 \times 3041 = \mathbf{6082} \text{ cm}^2$ .

167. The math club earned  $12 \times 3.50 + 13 \times 2.75 + 21 \times 4.00 + 11 \times 2.75 + 16 \times 3.50 = 42 + 35.75 + 84 + 30.25 + 56 = \mathbf{\$248}$ .

168. Since  $2026 \div 9 = 225 \frac{1}{9}$ , the integer  $N$  could have 225 digits of 9 and 1 digit of 1. That's **226** digits, which is the minimum.

169. The value of  $6 \pm 4$  is  $6^2 \div 4 = 36 \div 4 = \mathbf{9}$ .

170. The 192-page book could be divided into  $192 \div 8 = 24$  sets of 8 pages, so it will take Sam  $24 \times 10 = 240$  minutes or  $240 \div 60 = \mathbf{4}$  hours to read the book.