

Math Competition Strategy Solutions

- Underline the question and the form of the answer

If you aren't sure where to start with a problem, try one of these strategies:

- Does prime factoring the numbers help?
 - Can you draw a picture?
 - Can you make triangles out of it?
 - Are the triangles equilateral? 30-60-90? 45-45-90? Are there Pythagorean triples?
 - Can you solve a simpler similar problem?
 - Use numbers instead of variables; use smaller numbers,
 - Can you make an organized list or table?
 - Can you work backwards?
-

- Does prime factoring the numbers help?

1) How many more factors does 2025 have than 2026?

How many factors does 2025 have? Prime factor $2025 = 3^4 * 5^2$

Number of factors of 2025 = $(4+1) * (2+1) = 15$

How many factors does 2026 have? Prime factor $2026 = 2^1 * 1013^1$

Number of factors of 2026 = $(1+1) * (1+1) = 4$

$$15 - 4 = \underline{11}$$

	2	2026
1013		1013
		1

	3	2025
	3	675
	3	225
	3	75
	5	25
	5	5
		1

- Does prime factoring the numbers help?

2) What is the positive difference between the Greatest Common Factor (GCF) of 20 and 50 and the Least Common Multiple (LCM) of 20 and 50?

Prime factor $20 = 2^2 * 5^1$

Prime factor $50 = 2^1 * 5^2$

	2	20
	2	10
	5	5
		1

	2	50
	5	25
	5	5
		1

GCF is largest number that is a factor of both. It's the largest number of prime factors that are common to both numbers. $GCF \text{ of } 20 \text{ and } 50 = 2^1 * 5^1 = 10$

LCM is smallest number that has both 20 and 50 as factors. It includes all the prime factors of both numbers.

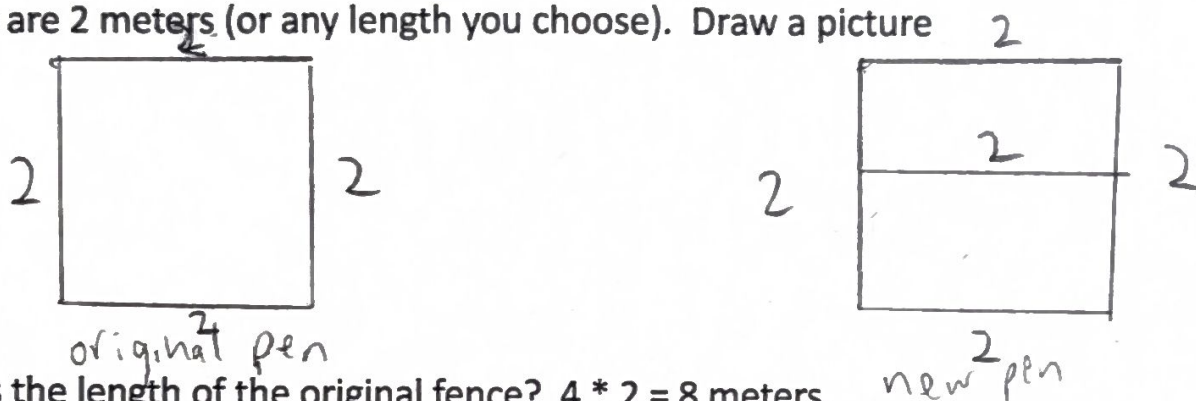
$LCM \text{ of } 20 \text{ and } 50 = 2^2 * 5^2 = 100$

Difference of LCM and GCF = $100 - 10 = \underline{90}$.

- Can you draw a picture?
- Can you solve a simpler similar problem?
 - Use numbers instead of variables; use smaller numbers

3) A square pen that is surrounded by a fence is divided into two rectangular regions by adding a new fence that connects the midpoints of two opposite sides. By what percent has the total length of fencing been increased?

The problem doesn't state the length of the sides. To make the problem easier, you can assume the side lengths are 2 meters (or any length you choose). Draw a picture



What is the length of the original fence? $4 * 2 = 8$ meters.

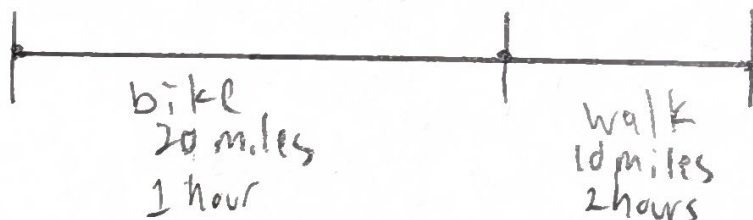
How much fence has been added? 2 meters

By what percent has the total length of fencing been increased? $(2 \text{ meters}) / (8 \text{ meters}) = 0.25 = \underline{25\%}$

- Can you draw a picture?
- Can you solve a simpler similar problem?
 - Use numbers instead of variables; use smaller numbers

4) A cyclist has completed two-thirds of his route when he gets a flat tire. He walks the rest of the way and spends twice as much time walking as he did riding. How many times as fast does he ride his bicycle as he walks?

Draw a picture to better understand the problem.



The problem doesn't state how far he rode, so to make it easier, assume his total route is 30 miles. That means he rode for 20 miles and walked for 10 miles. The problem doesn't say how long he spent riding, so to make it easier, assume he rode for 1 hour. That means he walked for 2 hours.

	Bike	Walk	Total
Distance	20 miles	10 miles	30 miles
Time	1 hour	2 hours	
Speed	20 miles/hour	5 miles/hour	

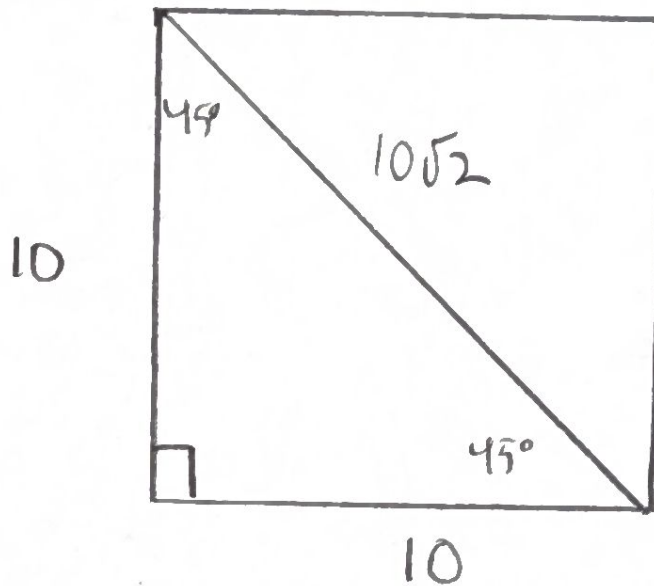
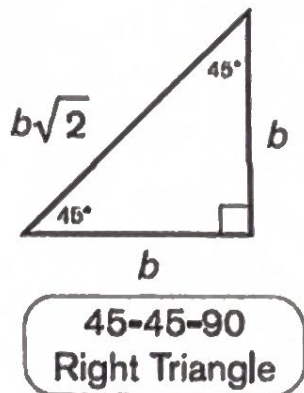
How many times as fast does he ride his bicycle as he walks?

$(20 \text{ miles/hour}) / (5 \text{ miles/hour}) = \underline{4}$

- Can you draw a picture?
- Can you make triangles out of it?
 - Are the triangles equilateral? 30-60-90? 45-45-90? Are there Pythagorean triples?

5) A junior high math coach gives mathletes the opportunity to throw paper airplanes into a trash can. The room they practice in is 10 meters by 10 meters. The mathletes stand in a corner of the room and throw the paper airplane to a trashcan in the opposite corner of the room. How far do the mathletes have to throw the paper airplane to reach the trashcan?

Draw a picture to better understand the problem.

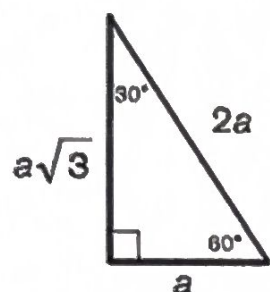


It's a 45-45-90 triangle. The sides are the length of the side of the room and the distance of the throw is the hypotenuse. Distance of the throw = $10\sqrt{2}$

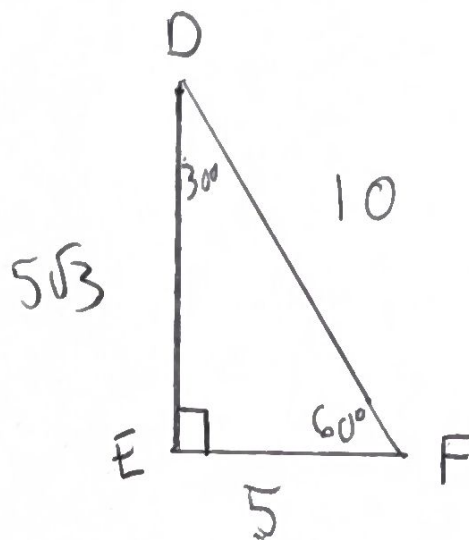
- Can you draw a picture?
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6) Triangle DEF is a right triangle. Angle FDE is 30 degrees and angle FED is 90 degrees. Side EF is 5 inches. How long is side FD? How long is side ED?

Draw a picture to better understand the problem.



30-60-90
Right Triangle

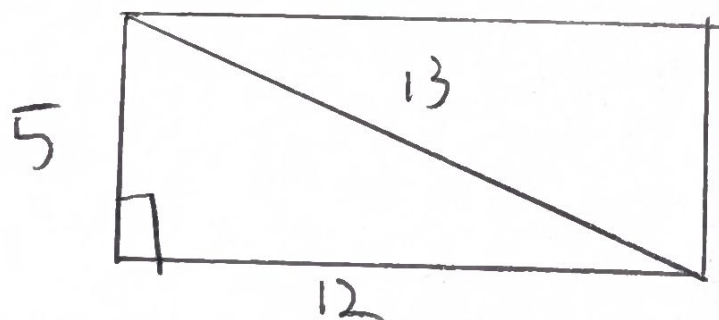


In a 30-60-90 triangle, the hypotenuse is twice as long as the side opposite the 30 degree angle. Side DF = 10 inches and ED = $5\sqrt{3}$.

- Can you draw a picture?
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 - Are the triangles equilateral? 30-60-90? 45-45-90? Are there Pythagorean triples?

7) One side of a rectangle is 5 inches. The diagonal is 13 inches. What is the length of the other side?

Draw a picture to better understand the problem.

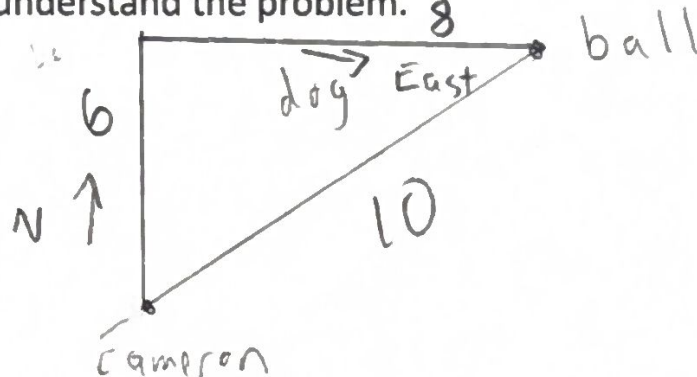


The diagonal and sides of a rectangle form a right triangle. If one side is 5, and the hypotenuse is 13, then it must be a 5-12-13 triangle. 5-12-13 is a Pythagorean triple. The other side is **12 inches**.

- Can you draw a picture?
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8) Cameron was standing next to their dog in the backyard. Cameron threw a ball 6 yards north. Cameron's dog ran to the ball, picked up the ball, then ran east 8 yards, put down the ball, and lied down to take a nap. What is the shortest distance for Cameron to walk to retrieve the ball?

Draw a picture to better understand the problem. 8



Based on the picture, we see that the shortest distance is the hypotenuse of a right triangle with sides of 6 yards and 8 yards. 3-4-5 is a Pythagorean triple. If we notice that $6 = 3 * 2$ and $8 = 4 * 2$, then we can quickly determine that the hypotenuse is $5 * 2 = \underline{10 \text{ yards}}$

- Can you make an organized list or table?

9) If the permutations of the letters in the word SURE are numbered 1 through 24 in alphabetical order, what number is RUSE? (For example, 1 is ERSU and 2 is ERUS)

Make an organized list of the permutations in alphabetical order

- 1- ERSU
- 2 - ERUS
- 3- ESRU
- 4 - ESUR
- 5 - EURS
- 6 - EUSR
- 7 - RESU
- 8 - REUS
- 9 - RSEU
- 10 - RSUE
- 11 - RUES
- 12 - RUSE