Warm-Up 5

71. A Pythagorean triple is a set of 3 integers that satisfy the Pythagorean theorem, $a^2 + b^2 = c^2$. If these numbers are the side lengths of a triangle, then we know that it is a right triangle. The most common example is the 3-4-5 triple, since $3^2 + 4^2 = 5^2$. To find a Pythagorean triple with the numbers 20 and 25, we multiply the 3-4-5 triple by 5, which gives us the lengths 15, 20 and 25. Since AB = 20 cm and AC = 25 cm, side BC must equal 15 cm. These 3 side lengths form a right triangle because $15^2 + 20^2 = 25^2$. Since the legs of a right triangle are the 2 shorter sides, the legs must be 15 cm and 20 cm, not 20 and 25, since $20^2 + 25^2 = 1025$, which is not a perfect square. So AB and BC are the legs and are perpendicular to each other, and we can calculate the area as 1/2 × 15 × 20 = **150** cm^2.

72. It is helpful to think of the ratio x/y as 3/1, so we think of it as 3 + 1 = 4 parts. The sum of x and y is 20, so each part is 20 ÷ 4 = 5. The numbers must be $x = 15$ and $y = 5$. The value of $x - y$ is 15 − 5 = **10**.

73. To find the least numeric palindrome divisible by 12, we list multiples of 12 in order until we reach one that reads the same forwards and backwards. The first such number is **252**.

74. Given that the real heptadecagon has a perimeter of 510 feet, we can calculate that each side of the real heptadecagon is 510 ÷ 17 = 30 feet. We are told that one side of the model heptadecagon is 2 inches, which is 1/6 of a foot. The scale factor of the model must be 1/6 ÷ 30 = 1/180, which is to say that all linear measurements on the actual Gauss Monument must be 180 times as long as the corresponding measurements on the scale model. Since the scale model is 3 feet tall, the real monument must be 180 × 3 feet = **540** feet tall.

75. The base ten value of 123_{four} is $1 \times 4^2 + 2 \times 4 + 3 = 16 + 8 + 3 = 27$, and the base ten value of 1234_{five} is $1 \times 5^3 + 2 \times 5^2 + 3 \times 5 + 4 = 125 + 50 + 15 + 4 = 194$. The desired sum is 27 + 194 = **221**.

76. The sum of the first 5 positive perfect squares is 1 + 4 + 9 + 16 + 25 = 55. This is also the sum of the first 10 positive integers, since 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55, so the value of n is **10**.

77. We will consider each of the possible tens digits and decide how many three-digit integers meet the conditions. For a tens digit of 0, there is only 1 three-digit integer and that's 101. For a tens digit of 1, there are 2 three-digit integers: 210 and 212. There are 4 three-digit integers with a tens digit of 2: 121, 123, 321 and 323. For tens digits of 3 through 8, there are likewise 4 three-digit integers. Finally, there is just 1 three-digit integers with a tens digit of 9 and that's 898. In all, there are 1 + 2 + 7 × 4 + 1 = **32** integers.

78. Cory can frost 2 dozen cupcakes in 14 minutes, which is a rate of 1 dozen every 7 minutes or 12/7 cupcakes per minute. Dory can frost 3 dozen cupcakes in 15 minutes, which is a rate of 1 dozen every 5 minutes or 12/5 cupcakes per minute. Together, they can frost 12/7 + 12/5 = 60/35 + 84/35 = 144/35 cupcakes per minute or 144 cupcakes in 35 minutes. That's 12 dozen cupcakes, so 1 dozen cupcakes would take 35/12 minutes, which is just under 3 minutes. At Cory's rate of 12/7 cupcakes per minute, he will put frosting on 12/7 × 35/12 = **5** cupcakes.

79. The mean of the numbers is (2 + 5 + 11 + 14 + 23) ÷ 5 = 55 ÷ 5 = 11, and the median of the numbers is 11. The absolute difference between the mean and the median is |11 − 11| = **0**.

80. There are 4! = 24 possible four-digit passcodes using those numbers, so the probability that Garrett guesses the correct one on his first try is **1/24**.