Warm-Up 4

61. Amy can pick any of $3 \times 3 \times 3 = 27$ outfits, so 27 will be our working denominator to put pieces together, though we may need to reduce it in the final answer. Rather than count the number of ways that at least 2 pieces of her outfit are the same color, we can more easily count the complement of this condition, which is the number of ways that all 3 pieces are different colors. Amy can pick one of 3 colors for the headband, then either of the 2 remaining colors for the sweater, and then she has to pick the 1 remaining color for the skirt, so there are $3 \times 2 \times 1 = 6$ outfits with no matching colors. This means there are $27 - 6 = 21$ outfits with at least 2 pieces that are the same color, so the desired probability is 21/27 or **7/9**.

62. Since $6 \times 6 = 36$, all powers of numbers ending in 6 end in 6. Similarly, all powers of numbers ending in 5 end in 5, since $5 \times 5 = 25$. The units digit of $2026^{2025} + 2025^{2026}$ will be equal to the units digit of $6 + 5 = 11$, which is **1**.

63. Let's say that the cube has an edge length of x. There are 12 congruent edges on a cube, so the total length of the edges is $12x$. There are 6 congruent faces on the cube, so the total surface area of the cube is $6x^2$. Since these two quantities are equal on this particular cube, we can solve the equation $12x = 6x^2$ for x. In general, we should be careful about dividing both sides of an equation by a variable since we might inadvertently divide by zero. In this case, however, we can be sure that the cube has an edge length greater than zero. We can divide both sides of our equation by x and we get $12 = 6x$, so $x = 2$. We now know the cube has an edge length of 2 feet and we can calculate that the volume is $2 \times 2 \times 2 =$ **8** ft^3.

64. Let's imagine that 29 of Professor Plum's students have actually brought plums to school and have redistributed then so that everyone has the same number of plums. This is the average number of plums. Imagine further that Scarlet arrives late with 100 plums. When Scarlet gives 1 plum to each of the other 29 students, the $100 - 29 = 71$ plums that she keeps for herself just happens to be the same number that everyone else has. This is the new average. Although the scores on the final exam are not actually redistributed like we imagined with the plums, the concept of an average is the number you would get if you did redistribute. Therefore, the correct class average on the final exam must be **71**. Alternatively, we can use algebra and let x equal the first average. Then $29x$ is the total score without Scarlet and $29x + 100$ is the total score with Scarlet. If we divide the total with Scarlet by 30, we get one more than the old average, which we write as $x + 1$. Our equation can be solved as follows: $(29x + 100)/30 = x + 1 \rightarrow 29x + 100 = 30(x + 1) \rightarrow 29x + 100 = 30x + 30 \rightarrow 100 = x + 30 \rightarrow x = 70$. So the first average without Scarlet was 70 and the correct class average with Scarlet is $70 + 1 =$ **71**.

65. There are $2 \times 2 = 4$ equally likely possible outcomes for the coin that is tossed twice. We will list the outcomes as HH, HT, TH and TT. Outcome HH means that the coin lands on heads twice and the counter would end up at S. Outcomes HT and TH result in the counter returning to square T. And outcome TT leaves the counter on square M. In 2 out of 4 outcomes, the counter returns to square T, so the probability is 2/4 or **1/2**.

66. Let's try some small values of n and work our way up. If $n = 2$, we get $2! = 1 \times 2 = 2$, which is not divisible by $2^2 = 4$. If $n = 3$, we get $3! = 1 \times 2 \times 3 = 6$, which is not divisible by $3^2 = 9$. If $n = 4$, we get $4! = 1 \times 2 \times 3 \times 4 = 24$, which is not divisible by $4^2 = 16$. If $n = 5$, we get $5! = 1 \times 2 \times 3 \times 4 \times 5 = 120$, which is not divisible by $5^2 = 25$. Finally, if $n = 6$, we get $6! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$, which is in fact divisible by $6^2 = 36$. So our answer is $n =$ **6**.

67. The original flight was expected to be 4:10 – 2:40 = 1:30 hours long, which is 1 hour and 30 minutes. If we add this amount of time to the 3:15 departure time, we get an arrival time of 3:15 + 1:30 = 4:45 p.m. Micah's connecting flight will leave 5:05 – 4:45 = 0:20 hour or **20** minutes later.

68. To count the number of isosceles right triangles with legs of 1 unit, we consider all four orientations. There are 15 triangles with the right angle at the bottom-left (◣), 10 triangles with the right angle at the bottom-right (◢), 10 triangles with the right angle at the top-left (◤), and 10 triangles with the right angle at the top-right (◥). Thus, there are $15 + 10 + 10 + 10 =$ **45** isosceles right triangles, with legs of 1 unit, in the array.

69. If we triple both sides of the first equation, we get $6a + 3b =$ **39**.

70. Arlo counts 100, 93, 86, etc. until he gets to $100 - 15 \times 7 = 100 - 105 = -5$. He then starts adding 6, giving him 1, 7, 13, etc. until he gets to $1 + 17 \times 6 = 1 + 102 =$ **103**.