Team 1

Assume the interior of the box is 1 foot = 12 inches tall, 14 inches wide and 16 inches deep. We could stack the 2-inch cubes 12 ÷ 2 = 6 high and 14 ÷ 2 = 7 wide, creating a layer of 6 × 7 = 42 cubes. With an interior depth of 16 inches, we could fill the box with 16 ÷ 2 = 8 such layers of 42 cubes, for a total of 8 × 42 = **336** cubes.

Team 2

Ms. Short's expenses are $1625 + 3 × $975 = $4550 each month. That means she has $4875 – $4550 = $325 available for entertainment. Since entertainment will account for 325/4875 = 1/15 of the budget, the sector of the graph for that category must have a central angle that is 1/15 of the 360 degrees in the full circle. That's 1/15 × 360 = **24** degrees.

Team 3

Since all 20 legs must belong to the cows, who have four legs each, there must be 20 ÷ 4 = 5 cows. Next, since all 17 heads must belong to the cows and octopi, all of which have one head each, there must be 17 − 5 = 12 octopi. Finally, the 196 tentacles must belong to the octopi, who have eight tentacles, and the jellyfish, who each have 20 tentacles. So, there must be (196 − 12 × 8) ÷ 20 = 5 jellyfish. Thus, Farmer Phil has 5 + 12 + 5 = **22** animals.

Team 4

For 2-point shots, Lisette made 67% ≈ 2/3 of 15 attempts, or 2/3 × 15 = 10 shots; Sara made 80% = 4/5 of 10 attempts, or 4/5 × 10 = 8 shots; Jen made 75% = 3/4 of 4 attempts, or 3/4 × 4 = 3 shots; and Tai made 33% ≈ 1/3 of 6 attempts, or 1/3 × 6 = 2 shots. That's a total of 10 + 8 + 3 + 2 = 23 shots made worth 2 points. For 3-point shots, Lisette made 40% = 2/5 of 5 attempts, or 2/5 × 5 = 2 shots; Sara made 50% = 1/2 of 2 attempts, or 1/2 × 2 = 1 shot; Jen did not make the 1 shot she attempted; and Tai made 50% = 1/2 of 2 attempts, or 1/2 × 2 = 1 shot. That's a total of 2 + 1 + 0 + 1 = 4 shots made worth 3 points. So, of the 45 shots attempted, 23 + 4 = 27 shots were made. The percent of shots that were successful is 27/45 = 3/5 = **60**%.

Team 5

There are two cases in which a power of $(x – 2)$ is equal to 1: either $x – 2 = 1$, or $x – 2 = –1$ and $25 – x^2$ is even. Then there is the case in which the exponent is $25 – x^2 = 0$. Let's examine all three cases.

Case 1: If $x – 2 = 1$, then $x = 3$. So, we have $(x – 2)^{25 – x^2} = (3 – 2)^{25 – 3^2} = 1^{16} = 1$. Thus, $x = 3$ is a solution.

Case 2: If $x – 2 = –1$, then $x = 1$. So, we have $(x – 2)^{25 – x^2} = (1 – 2)^{25 – 1^2} = (–1)^{24} = 1$. Thus, $x = 1$ is a solution.

Case 3: If $25 – x^2 = 0$, then $x^2 = 25$ and $x = ±5$. So, we have $(x – 2)^{25 – x^2} = (5 – 2)^{25 – 5^2} = 3^0 = 1$ and $((–5) – 2)^{25 – (–5)^2} = (–7)^0 = 1$. That means both $x = 5$ and $x = –5$ are solutions.

That gives us a total of **4** integers that are solutions.

Team 6

The formula for the volume of a sphere is $V = (4/3)\pi r^3$. There are three tennis balls with radius 4 cm, so the total volume of the tennis balls is $3 \times (4/3) \times \pi \times 4^3 = 4^4 \times \pi = 256\pi$ cm^3. The formula for the volume of a cylinder is $V = \pi r^2 h$. The cylinder has a radius of 4 cm and a height of 3 × 8 cm = 24 cm. The volume of the cylinder is $\pi \times 4^2 \times 24 = 384\pi$ cm^3. The empty space in the can is 384π – 256π = (384 – 256)π = **128π** cm^3.

Team 7

Caleb took 1/3 of the jellybeans in the bag and then ate 1/2 of those, so he actually had 1/6 of the entire original bag of jellybeans left. Zara took 1/2 of the jellybeans in the bag and then ate 1/3 of those, leaving 1/2 – (1/3 × 1/2) = 1/2 – 1/6 = 1/3 of the entire original bag of jellybeans left. Between the two of them, they now have 45 jellybeans, which must be 1/6 + 1/3 = 1/6 + 2/6 = 3/6 = 1/2 of the entire original bag of jellybeans. Thus, the total number of jellybeans that were originally in the bag was 45 × 2 = 90 jellybeans. Between Caleb and Zara, they originally took 1/3 + 1/2 = 2/6 + 3/6 = 5/6 of the entire original bag, leaving just 1/6 of the bag for Kris. So, Kris ate (1/6) × 90 = **15** jellybeans.

Team 8

Determine the positive integer divisors of 175 by first factoring to find 175 = 5 × 5 × 7. Thus, the positive integer divisors of 175, besides 1, are 5, 7, 25, 35 and 175. Only two of these values are multiples of 7, namely 35 and 175, and thus would be adjacent to 7 around the circle. So, the sum of these two integers is 175 + 35 = **210**.

Team 9

The number of five-digit integers where none of the digits are repeated is 5! = 120. The number of different arrangements for a 5-digit integer containing two 2s (i.e. 22_ _ _, 2_2_ _, etc.) is 5!/2!3! = (5 × 4)/2 = 10 arrangements. For each arrangement, we have to fill in the other three digits. There are 4 choices for the first digit, 3 choices for the second digit and 2 choices for the last digit. That's 4 × 3 × 2 = 24 combinations of digits for each arrangement, giving us a total of 24 × 10 = 240 such 5-digit integers containing two 2s. The number of different arrangements for a 5-digit integer containing three 2s is 5!/3!2! = (5 × 4)/2 = 10 arrangements. For the remaining two digits, we have 4 choices for the first digit and 3 choices for the second digit. That's 4 × 3 = 12 combinations of digits for each arrangement, giving us a total of 12 × 10 = 120 such 5-digit integers containing three 2s. The number of different arrangements for a 5-digit integer containing four 2s is 5!/4!1! = 5 arrangements. For each arrangement, there are 4 choices for the digit that is not a 2, giving us a total of 5 × 4 = 20 such 5-digit integers containing four 2s. Finally, there is only 1 arrangement for the 5-digit number containing five 2s. So, in total, there are 120 + 240 + 120 + 20 + 1 = **501** such integers.

Team 10

We have 3 + 4 = 7 non-yellow balls, so they must all be drawn first. There is a total of 3 + 4 + 5 = 12 balls. So, the probability that the 7 non-yellow balls are drawn first is (7 × 6 × 5 × 4 × 3 × 2 × 1)/(12 × 11 × 10 × 9 × 8 × 7 × 6) = **1/792**.