MAA AMC

American Mathematics Competitions

MAA American Mathematics Competitions

37th Annual

AMC 8

Tuesday, January 18, 2022 through Monday, January 24, 2022

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR COMPETITION MANAGER TELLS YOU TO BEGIN.
2. This is a 25-question multiple-choice competition. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the answer sheet with a #2 pencil. Check blackened answers for accuracy and erase errors completely. Only answers that are properly marked on the answer sheet will be scored.
4. SCORING: You will receive 1 point for each correct answer, 0 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only blank scratch paper, rulers, and erasers are allowed as aids. Prohibited materials include calculators, smartwatches, phones, computing devices, compasses, protractors, and graph paper. No problems on the competition will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the competition, your competition manager will ask you to record your name and other information on the answer sheet.
8. You will have 40 minutes to complete the competition once your competition manager tells you to begin.
9. When you finish the competition, sign your name in the space provided on the answer sheet.

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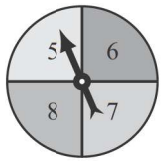
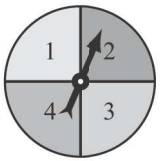
12. The arrows on the two spinners shown below are spun. Let the number N equal 10 times the number on Spinner A, added to the number on Spinner B. What is the probability that N is a perfect square number?

(A) $\frac{1}{16}$ **(B)** $\frac{1}{8}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{3}{8}$ **(E)** $\frac{1}{2}$



Spinner A



Spinner B

13. How many positive integers can fill the blank in the sentence below?

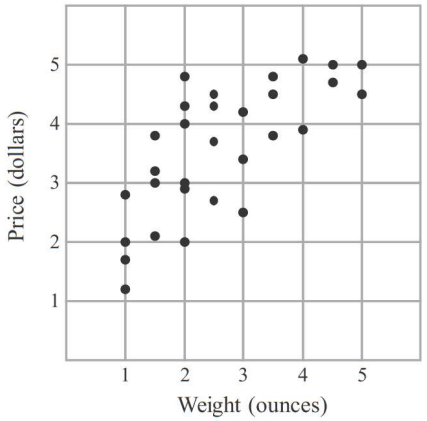
"One positive integer is ____ more than twice another, and the sum of the two numbers is 28."

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

14. In how many ways can the letters in BEEKEEPER be rearranged so that two or more Es do not appear together?

(A) 1 **(B)** 4 **(C)** 12 **(D)** 24 **(E)** 120

15. Laszlo went online to shop for black pepper and found thirty different black pepper options varying in weight and price, shown in the scatter plot below. In ounces, what is the weight of the pepper that offers the lowest price per ounce?



(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

16. Four numbers are written in a row. The average of the first two is 21, the average of the middle two is 26, and the average of the last two is 30. What is the average of the first and last of the numbers?

(A) 24 **(B)** 25 **(C)** 26 **(D)** 27 **(E)** 28

17. If n is an even positive integer, the *double factorial* notation $n!!$ represents the product of all the even integers from 2 to n. For example, $8!! = 2 \cdot 4 \cdot 6 \cdot 8$. What is the units digit of the following sum?

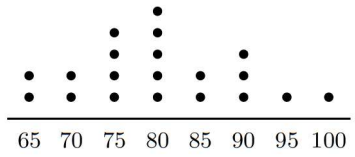
$$2!! + 4!! + 6!! + \cdots + 2018!! + 2020!! + 2022!!$$

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

18. The midpoints of the four sides of a rectangle are $(-3, 0)$, $(2, 0)$, $(5, 4)$, and $(0, 4)$. What is the area of the rectangle?

(A) 20 **(B)** 25 **(C)** 40 **(D)** 50 **(E)** 80

19. Mr. Ramos gave a test to his class of 20 students. The dot plot below shows the distribution of test scores.



Later Mr. Ramos discovered that there was a scoring error on one of the questions. He regraded the tests, awarding some of the students 5 extra points, which increased the median test score to 85. What is the minimum number of students who received extra points?

(Note that the *median* test score equals the average of the 2 scores in the middle if the 20 test scores are arranged in increasing order.)

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

20. The grid below is to be filled with integers in such a way that the sum of the numbers in each row and the sum of the numbers in each column are the same. Four numbers are missing. The number x in the lower left corner is larger than the other three missing numbers. What is the smallest possible value of x?

-2	9	5
		-1
x		8

(A) -1 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 9

21. Steph scored 15 baskets out of 20 attempts in the first half of a game, and 10 baskets out of 10 attempts in the second half. Candace took 12 attempts in the first half and 18 attempts in the second. In each half, Steph scored a higher percentage of baskets than Candace. Surprisingly they ended with the same overall percentage of baskets scored. How many more baskets did Candace score in the second half than in the first?

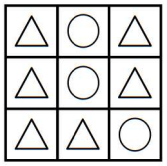
	First Half	Second Half
Steph	$\frac{15}{20}$	$\frac{10}{10}$
Candace	$\frac{\square}{12}$	$\frac{\square}{18}$

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

22. A bus takes 2 minutes to drive from one stop to the next, and waits 1 minute at each stop to let passengers board. Zia takes 5 minutes to walk from one bus stop to the next. As Zia reaches a bus stop, if the bus is at the previous stop or has already left the previous stop, then she will wait for the bus. Otherwise she will start walking toward the next stop. Suppose the bus and Zia start at the same time toward the library, with the bus 3 stops behind. After how many minutes will Zia board the bus?

(A) 17 **(B)** 19 **(C)** 20 **(D)** 21 **(E)** 23

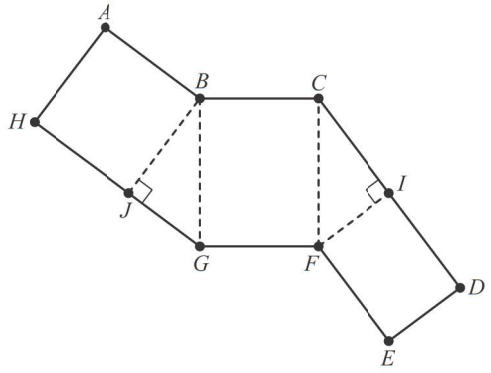
23. A △ or ○ is placed in each of the nine squares in a 3-by-3 grid. Shown below is a sample configuration with three △s in a line.



How many configurations will have three △s in a line and three ○s in a line?

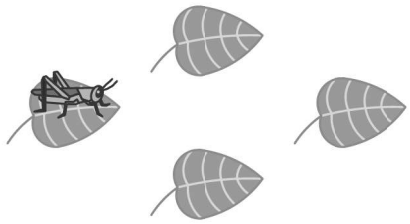
(A) 39 **(B)** 42 **(C)** 78 **(D)** 84 **(E)** 96

24. The figure below shows a polygon $ABCDEFGH$, consisting of rectangles and right triangles. When cut out and folded on the dotted lines, the polygon forms a triangular prism. Suppose that $AH = EF = 8$ and $GH = 14$. What is the volume of the prism?



(A) 112 **(B)** 128 **(C)** 192 **(D)** 240 **(E)** 288

25. A cricket randomly hops between 4 leaves, on each turn hopping to one of the other 3 leaves with equal probability. After 4 hops, what is the probability that the cricket has returned to the leaf where it started?



(A) $\frac{2}{9}$ **(B)** $\frac{19}{80}$ **(C)** $\frac{20}{81}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{7}{27}$

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Scores and official competition solutions will be sent to your competition manager who can share that information with you.

For more information about the MAA American Mathematics Competitions program and our other competitions, please visit maa.org/amc.

Questions and comments about this competition should be sent to:

amcinfo@maa.org

or

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The problems and solutions for this AMC 8 were prepared by the MAA AMC 8 Editorial Board under the direction of:
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MAA Partner Organizations

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DO NOT OPEN UNTIL COMPETITION DAY.

The AMC 8 may be administered from

Tuesday, January 18, 2022 through Monday, January 24, 2022.

****Administration on an earlier date will disqualify your school's results.****

- All the information needed to administer this competition is contained in the AMC 8 Teacher's Manual. PLEASE READ THE MANUAL BEFORE TUESDAY, JANUARY 18, 2022.
- Answer sheets must be returned to the MAA AMC office within 24 hours of the competition administration. Use an overnight or 2-day shipping service, with a tracking number, to guarantee timely arrival of these answer sheets. FedEx, UPS, or USPS overnight are strongly recommended.
- The publication, reproduction, or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.