

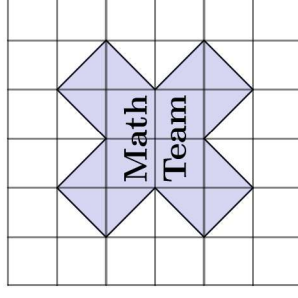
2022 AMC 8 Solutions

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<https://live.poshenloh.com/past-contests/amc8/2022/solutions>



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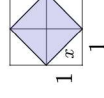
1. The Math Team designed a logo shaped like a multiplication symbol, shown below on a grid of 1-inch squares. What is the area of the logo in square inches?



- ☒ A 10
☐ B 12
☐ C 13
☐ D 14
☐ E 15

Solution(s):

Let's first consider the following:



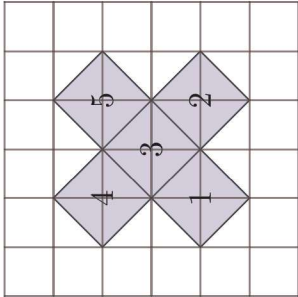
Using the Pythagorean theorem, we know how to solve for x relatively easily:

$$\begin{aligned}a^2 + b^2 &= c^2 \iff 1^2 + 1^2 = x^2 \\ \iff x &= \sqrt{2}\end{aligned}$$

With that in mind, let's try and find the area of the purple square:

$$A = x^2 = (\sqrt{2})^2 = 2$$

We now know that the area of this slanted, small square is equal to 2. With that in mind, let's look at the full picture:



We can clearly see that there are 5 of these same small purple squares that make up the logo. We already know that the area of one of the small purple squares is equal to 2, so the total area A_t is:

$$A_t = 5(A) = 5(2) = 10$$

Thus, the answer is **A**.

2. Consider these two operations:

$$a \blacklozenge b = a^2 - b^2$$

$$a \star b = (a - b)^2$$

Compute the value:

$$(5 \blacklozenge 3) \star 6$$

- A-20
- B4
- C16
- D100
- E220

Solution(s):

Given the definitions of $\blacklozenge$ and $\star$, we can directly substitute these operations for said definitions to get:

$$\begin{aligned} (5 \blacklozenge 3) \star 6 &= (5^2 - 3^2) \star 6 \\ &= 16 \star 6 \\ &= (16 - 6)^2 \\ &= 100 \end{aligned}$$

Thus, the answer is **D**.

3. When three positive integers a , b , and c are multiplied together, their product is 100. Suppose $a < b < c$. In how many ways can the numbers be chosen?

- ☐ A 0
☐ B 1
☐ C 2
☐ D 3
☒ E 4

Solution(s):

We are given that $a < b < c$, $abc = 100$.

As such, we know that $\sqrt[3]{a^3}$

Considering $\sqrt[3]{100}$, we know that $4^3 = 64 < 100$ and $5^3 = 125 > 100$, and therefore, we can conclude that $a \geq 4$. As a is a positive integer, we have four possible values of a : 1, 2, 3, 4. As a is a factor of 100, we can eliminate the possibility of a being 3, and are now left with three cases: $a = 1, 2, 4$.

If $a = 1$, then $bc = 100$. Since $b < c$, we know $b^2 < bc = 100$ so $1 < b < 10$. Since b is a factor of 100 that satisfies this constraint, b must be either 2, 4, 5. This creates the solutions of (1, 2, 50), (1, 4, 25), or (1, 5, 20.)

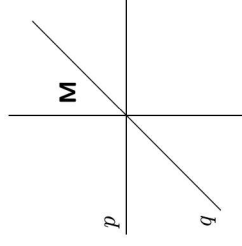
If $a = 2$, then $bc = 50$. Since $b < c$, we know $b^2 < bc = 50$ so $2 < b < 8$. Since b is a factor of 50 that satisfies this constraint, b must be 5. This creates the solution of (2, 5, 10.)

If $a = 4$, then $bc = 25$. Since $b < c$, we know $b^2 < bc = 25$ so $4 < b < 5$. This creates no integer solutions for b .

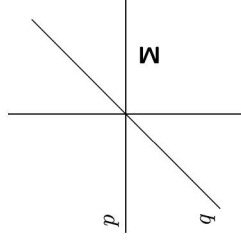
We therefore have 4 total solutions.

Thus, the answer is **A**.

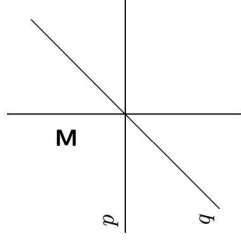
4. The letter **M** in the figure below is first reflected over the line q and then reflected over the line p . What is the resulting image?



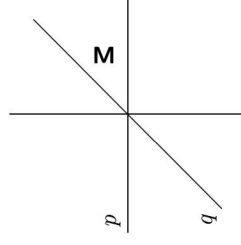
A

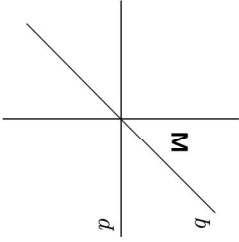


B

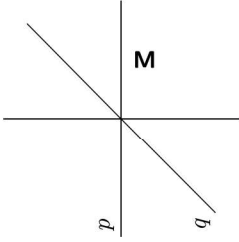


C





D



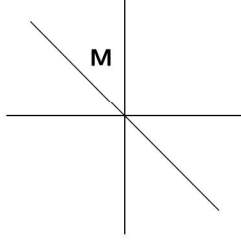
E

Solution(s):

First reflect

M

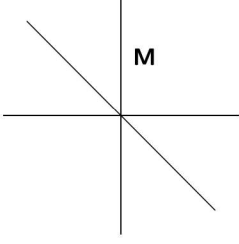
over the line q , from which we obtain the following:



Then reflect

M

over the line p , from which we obtain the following:



Thus, the answer is **E**.

5. Anna and Bella are celebrating their birthdays together. Five years ago, when Bella turned 6 years old, she received a newborn kitten as a birthday present. Today the sum of the ages of the two children and the kitten is 30 years. How many years older than Bella is Anna?

- A** 1
B 2
C 3
D 4
E 5

Solution(s):

If Bella was 6 five years ago, then she is 11 right now.

If the kitten was a newborn five years ago, then it is 5 right now.

Since the sum of all 3 ages is 30, Anna's age is

$$30 - 5 - 11 = 14$$

Since Anna is 14 and Bella is 11, she is 3 years older than Bella.

Thus, the answer is **C**.

6. Three positive integers are equally spaced on a number line. The middle number is 15 and the largest number is 4 times the smallest number. What is the smallest of these three numbers?

☐ A 4
☐ B 5
☒ C 6
☐ D 7
☐ E 8

Solution(s):

Since one of the outer number is 4 times the other, we can make one of the numbers x and the other be $4x$.

Since the numbers are equally spaced, the middle number is the average of the outer two.

This means the average of x and $4x$ is 15, so $\frac{x + 4x}{2} = 15$. This means $5x = 30$, so $x = 6$. Our outer numbers therefore are 6, 24, so the smaller one is 6.

Thus, the answer is **C**.

7. When the World Wide Web first became popular in the 1990s, download speeds reached a maximum of about 56 kilobits per second. Approximately how many minutes would the download of a 4.2-megabyte song have taken at that speed? (Note that there are 8000 kilobits in a megabyte.)

☐ A 0.6
☒ B 10
☐ C 1800
☐ D 7200
☐ E 36000

Solution(s):

Given our definition of a megabyte as 8000 kilobits, we know 4.2 megabytes is $8000 \cdot 4.2$ kilobits. This can be rewritten as $(8000 \cdot \frac{3}{40}) \cdot (4.2 \cdot \frac{40}{3})$ kilobits which leads to us having $600 \cdot 56$ kilobits. Since we know there are 56 kilobits per second, we have 600 seconds. This leads us to the answer of 10 minutes.

Thus, the answer is **B**.

8. What is the value of:

$$\frac{1}{3} \cdot \frac{2}{4} \cdot \frac{3}{5} \cdots \frac{18}{20} \cdot \frac{19}{21} \cdot \frac{20}{22}$$

A $\frac{1}{462}$

B $\frac{1}{231}$

C $\frac{1}{132}$

D $\frac{2}{213}$

E $\frac{1}{22}$

Solution(s):

Since every integer from 3 to 20 occurs once as a denominator and once as a numerator, they cancel each other out.

After canceling every number out, we have only 1 and 2 left as numerators and 21 and 22 left as denominators.

The remaining fraction is $\frac{1 \cdot 2}{21 \cdot 22}$. This simplifies to $\frac{2}{462} = \frac{1}{231}$

9. A cup of boiling water (212°F) is placed to cool in a room whose temperature remains constant at 68°F . Suppose the difference between the water temperature and the room temperature is halved every 5 minutes. What is the water temperature, in degrees Fahrenheit, after 15 minutes?

A 77

B 86

C 92

D 98

E 104

Solution(s):

The current difference is $212 - 68 = 144$.

Since we have 15 minutes while halving every 5 minutes, we halve the difference 3 times.

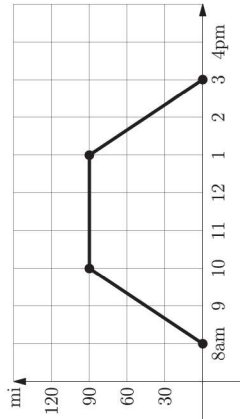
This means the difference is multiplied by $(\frac{1}{2})^3 = \frac{1}{8}$, so our new difference is

$$\frac{1}{8} \cdot 144 = 18. \text{ This makes our final temperature } 68 + 18 = 86.$$

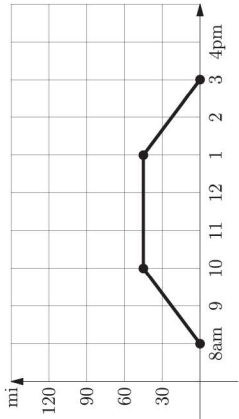
Thus, the answer is **B**.

10. One sunny day, Ling decided to take a hike in the mountains. She left her house at 8 a.m., drove at a constant speed of 45 miles per hour, and arrived at the hiking trail at 10 a.m. After hiking for 3 hours, Ling drove home at a constant speed of 60 miles per hour. Which of the following graphs best illustrates the distance between Ling's car and her house over the course of her trip?

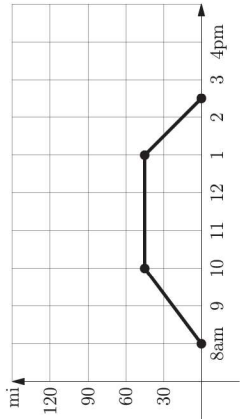
A



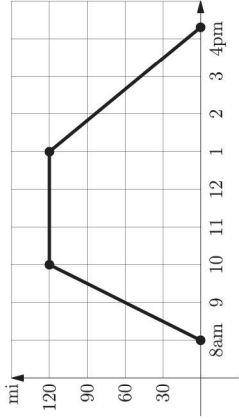
B



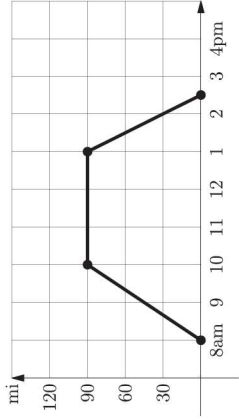
C



D



E



Solution(s):

She drives 45 miles per hour for the first 2 hours, so she travels 90 miles in the first two hours. This leaves choices *A*, *E*.

Since she hikes for 3 hours, she starts going back at 1pm.

She drives 90 miles at 60 mph, so she gets back in $\frac{90}{60} = 1.5$ hours, so she is back at 2:30.

Thus, the answer is **E**.

11. Henry the donkey has a very long piece of pasta. He takes a number of bites of pasta, each time eating 3 inches of pasta from the middle of one piece. In the end, he has 10 pieces of pasta whose total length is 17 inches. How long, in inches, was the piece of pasta he started with?

- ☐ A 34
- ☐ B 38
- ☐ C 41
- ☒ D 44
- ☐ E 47

Solution(s):

Since there are 10 pieces, there were 9 locations where a bite was made. Since we have 9 bites and 3 inches are removed per bite, a total of 27 inches were removed.

With 27 inches removed and 17 inches remaining, we know we started with $27 + 17 = 44$ inches.

Thus, the answer is **D**.