Answer Sheet

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1	480 square millimeters	1D344
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5	11 square meters	45B44
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Solutions

(1) **480 square millimeters ID: [1D344]**
If the initial triangle has an area of 120 mm^2 along with a height of 10 mm, then it must have a base of 24 mm. A similar triangle that has twice the height will have twice the base, 48 mm. Thus, the area of the similar triangle is $\frac{1}{2}(20 \text{ mm})(48 \text{ mm}) = \boxed{480} \text{ mm}^2$.

Alternatively, note that if the height in one dimension scaled $2\times$, then the area must have scaled $2^2\times$, giving us an answer of $4 \times 120 \text{ mm}^2 = \boxed{480} \text{ mm}^2$.

(2) **82 degrees ID: [45A4D]**
Since the sum of the angles in a triangle is 180°, the remaining angle of the triangle is $180° - (33° + 49°)$. Since angle A is supplementary to that angle, its measure is $33° + 49° = \boxed{82°}$.

(3) **75 degrees ID: [ABA34]**
Since $\triangle AED$ is equilateral, we know that $\angle ADE = 60°$, which implies that $\angle CDF$ is 30°. Also, as AC is a diagonal, it bisects right angle C, meaning that $\angle DCF = 45°$. Thus, $\angle CFD = 180° - 30° - 45° = 105°$.

Since $\angle EFC + \angle CFD$ must equal 180°, we know that $\angle EFC$ must be $180° - 105° = \boxed{75°}$.

(4) **64 square feet ID: [0D344]**
The side length of the square is equal to the diameter of the circle inscribed inside. The diameter is twice the radius, 2×4 ft $= 8$ ft, so the area of the square is $8 \text{ ft} \times 8 \text{ ft} = \boxed{64} \text{ ft}^2$.

(5) **11 square meters ID: [45B44]**
Let the sides of the triangle be a, b, and c where c is the length of the hypotenuse. We have that $c = 10$ and $a + b + c = 22$, so $a + b = 12$. From the Pythagorean Theorem, we also have that $a^2 + b^2 = c^2 = 10^2 = 100$. We wish to find the triangle's area, which is $\frac{1}{2}ab$.

Squaring both sides of $a + b = 12$ gives us an equation containing both what we have and what we want, which is $a^2 + 2ab + b^2 = 144$. Substituting $a^2 + b^2 = 100$, we get that $2ab = 44$. Dividing by 4 on both sides leaves us with the area of the triangle, $\boxed{11}$.

(6) $6\sqrt{2}$ **cm** **ID: [3D103]**

If we let $x =$ the side length of the triangle, then we can find the area of the triangle in terms of x and then set it equal to $16\sqrt{3}$ to find x. The base of the triangle has length x. To find the altitude, we notice that drawing an altitude splits the equilateral triangle into two $30-60-90$ triangles with the longest side having length x. Since the ratio of the side lengths of a $30-60-90$ triangle is $1 : \sqrt{3} : 2$, the altitude will have length $\frac{x\sqrt{3}}{2}$ and the area of the triangle will be $\frac{1}{2}x\left(\frac{x\sqrt{3}}{2}\right) = \frac{x^2\sqrt{3}}{4}$. Setting this equal to $16\sqrt{3}$, we have that $\frac{x^2\sqrt{3}}{4} = 16\sqrt{3}$.

Solving for x, we get that $x = 8$. Since the side length of the triangle is 8 and the square and triangle have equal perimeters, the square has a side length of $\frac{8\cdot 3}{4} = 6$. If we draw the diagonal of the square, we notice that it splits the square into two $45-45-90$ triangles with legs of length 6. A $45-45-90$ triangle has side length ratios of $1 : 1 : \sqrt{2}$, so the diagonal of the square has length $\boxed{6\sqrt{2}}$ cm.

(7) **18 square units** **ID: [20403]**

We first find the length of line segment FG. Since DC has length 6 and DF and GC have lengths 1 and 2 respectively, FG must have length 3. Next, we notice that DC and AB are parallel so $\angle EFG \cong \angle EAB$ because they are corresponding angles. Similarly, $\angle EGF \cong \angle EBA$. Now that we have two pairs of congruent angles, we know that $\triangle FEG \sim \triangle AEB$ by Angle-Angle Similarity.

Because the two triangles are similar, we have that the ratio of the altitudes of $\triangle FEG$ to $\triangle AEB$ equals the ratio of the bases. $FG : AB = 3 : 6 = 1 : 2$, so the the ratio of the altitude of $\triangle FEG$ to that of $\triangle AEB$ is also $1 : 2$. Thus, the height of the rectangle $ABCD$ must be half of the altitude of $\triangle AEB$. Since the height of rectangle $ABCD$ is 3, the altitude of $\triangle AEB$ must be 6. Now that we know that the base and altitude of $\triangle AEB$ are both 6, we know that the area of triangle AEB is equal to $\frac{1}{2}\text{base} \times \text{height} = (\frac{1}{2})(6)(6) = \boxed{18}$ square units.

(8) **8 square units** **ID: [C2A44]**

The base of the triangle is $5-1 = 4$ units, and the height of the triangle is $5-1 = 4$ units. Thus, the area of the triangle is $\frac{1}{2}(4)(4) = \boxed{8}$ square units.

(9) **35 inches ID: [03453]**

Since the smaller triangle has hypotenuse 5, we guess that it is a 3-4-5 triangle. Sure enough, the area of a right triangle with legs of lengths 3 and 4 is $(3)(4)/2 = 6$, so this works. The area of the larger triangle is $150/6 = 25$ times the area of the smaller triangle, so its side lengths are $\sqrt{25} = 5$ times as long as the side lengths of the smaller triangle. Therefore, the sum of the lengths of the legs of the larger triangle is $5(3 + 4) = \boxed{35}$.

Proof that the only possibility for the smaller triangle is that it is a 3-4-5 triangle: Let's call the legs of the smaller triangle a and b (with b being the longer leg) and the hypotenuse of the smaller triangle c. Similarly, let's call the corresponding legs of the larger triangle A and B and the hypotenuse of the larger triangle C. Since the area of the smaller triangle is 6 square inches, we can say

$$\frac{1}{2}ab = 6.$$

Additionally, we are told that the hypotenuse of the smaller triangle is 5 inches, so $c = 5$ and

$$a^2 + b^2 = 25.$$

Because $\frac{1}{2}ab = 6$, we get $ab = 12$ or $a = \frac{12}{b}$. We can now write the equation in terms of b. We get

$$\begin{aligned} a^2 + b^2 &= 25 \\ \left(\frac{12}{b}\right)^2 + b^2 &= 25 \\ 12^2 + b^4 &= 25b^2 \\ b^4 - 25b^2 + 144 &= 0. \end{aligned}$$

Solving for b, we get

$$b^4 - 25b^2 + 144 = (b - 4)(b + 4)(b - 3)(b + 3) = 0.$$

Since we said that b is the longer of the two legs, $b = 4$ and $a = 3$. Therefore, the triangle must be a 3-4-5 right triangle.

(10) **1 square inches ID: [AAA34]**

From the information given in the problem, we can deduce that the radius of circle Q is one inch. The height of the triangle, QY is also a radius, so the area of $\triangle XYZ$ is $\frac{1}{2}bh = \frac{1}{2}(2)(1) = \boxed{1 \text{ in}^2}$.