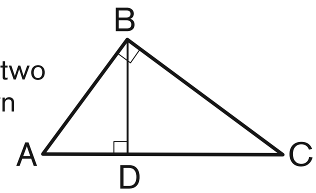
Triangles Stretch

A **cevian** (pronounced chay-vee-an) is a line segment drawn from a vertex of a triangle to a point on the opposite side (or an extension of the opposite side). Altitudes, medians and angle bisectors are examples of cevians.

ALTITUDES

An **altitude** of a triangle is a cevian drawn from a vertex and perpendicular to the line containing the opposite side. An altitude of a triangle can be inside the triangle, be outside the triangle, or coincide with a side of the triangle. Every triangle has three altitudes. They intersect at a point called the **orthocenter**, which is located inside an acute triangle, outside an obtuse triangle, and at the right-angle vertex of a right triangle.

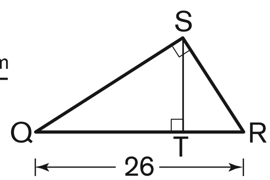


ALTITUDE THEOREM: The altitude perpendicular to the hypotenuse of a right triangle forms two triangles that are similar to each other and to the original triangle. In right triangle ABC, shown here, with altitude BD, $\triangle ABC \sim \triangle ADB \sim \triangle BDC$.

This theorem is also known as the **geometric mean theorem** because of the following properties:

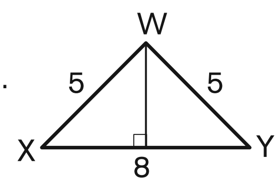
$\frac{AD}{BD} = \frac{BD}{CD} \Rightarrow BD = \sqrt{AD \times CD}$	$\frac{AD}{AB} = \frac{AB}{AC} \Rightarrow AB = \sqrt{AD \times AC}$	$\frac{CD}{BC} = \frac{BC}{AC} \Rightarrow BC = \sqrt{CD \times AC}$

21.________ cm



Right triangle QRS, shown here, has altitude ST. If QR = 26 cm and QT:RT = 9:4, what is ST?

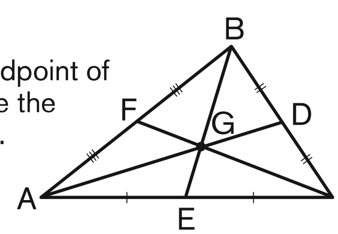
22.________ cm In isosceles triangle WXY, shown here, WX = WY = 5 cm and XY = 8 cm. What is the length of the altitude drawn perpendicular to side XY?



23.________ cm What is the arithmetic mean of the altitude lengths of an isosceles triangle with two sides of length 13 cm and one side of length 10 cm? Express your answer as a common fraction.

MEDIANS

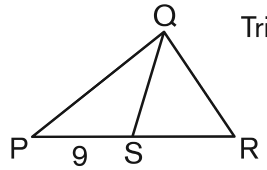
A **median** of a triangle is a cevian drawn from a vertex to the midpoint of the opposite side. Every triangle has three medians. They intersect inside the triangle at a point called the **centroid**.



CENTROID THEOREM: The centroid is two-thirds the distance from each vertex to the midpoint of the opposite side. In other words, the segment joining the vertex and centroid has twice the length of the segment joining the centroid and the midpoint of the opposite side. Triangle ABC, shown here, has medians AD, BE and CF, with the following properties:

AG = 2 × DG	BG = 2 × EG	CG = 2 × FG

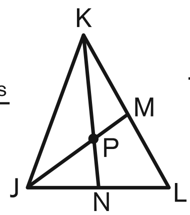
24. ________ cm



Triangle PQR, shown here, has area 24 cm^2. If segment QS is a median of triangle PQR and PS = 9 cm, what is the height of triangle PQR? Express your answer as a common fraction.

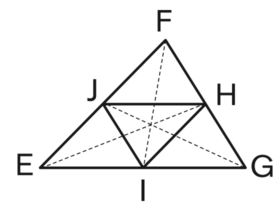
25. ________ Triangle ABC has medians AD, BE and CF with centroid G. If AG = $6x$ and DG = $5x - 8$, what is the value of x?

26. ________ units



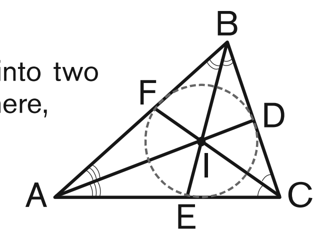
Triangle JKL has medians JM and KN with centroid P, as shown. If KN = $\frac{19x}{2}$ and NP = $\frac{5x}{2} + 2$, what is KP?

27. ________ cm^2 Triangle EFG has medians EH, FI and GJ. If EF = 25 cm, FG = 17 cm and EG = 26 cm, what is the area of triangle HIJ?



ANGLE BISECTORS

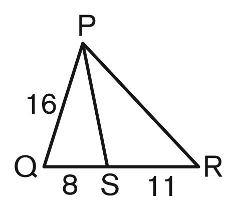
An **angle bisector** of a triangle is a cevian that divides a vertex angle into two congruent angles and intersects the opposite side. Every triangle has three angle bisectors. They intersect inside the triangle at a point called the **incenter**. The incenter is also the center of the triangle's **inscribed circle**, or **incircle**.



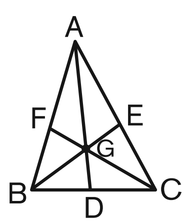
Angle Bisector Theorem: The angle bisector of a triangle divides the opposite side into two segments that are proportional to the other two sides of the triangle. Triangle ABC, shown here, has angle bisectors AD, BE and CF with the following proportions:

$$\frac{AB}{AC} = \frac{BD}{CD} \quad \Big| \quad \frac{AB}{BC} = \frac{AE}{CE} \quad \Big| \quad \frac{AC}{BC} = \frac{AF}{BF}$$

28. ________ cm Triangle PQR has angle bisector PS, as shown, with PQ = 16 cm, RS = 11 cm and QS = 8 cm. What is the perimeter of triangle PQR?



29. ________



Triangle ABC has angle bisectors AD, BE and CF, as shown, and AB = 10, BC = 7, AC = 11. What is the ratio of DG to AG? Express your answer as a common fraction.

30. ________ cm The figure shows an inscribed circle tangent to triangle ABC at points D, E and F on sides AB, BC and AC, respectively. If AB = 15 cm, BC = 14 cm and BD = 6 cm, what is AC?