Division **E**

Mathematical Olympiads for Elementary & Middle Schools

February 6, 2023

Contest **4**

Directions to Students: After all questions have been read by your PICO, you will have 30 minutes to complete this contest. You may not have a pen or pencil in your hand while the PICO reads the set of questions to the class. Calculators are not permitted. All work is to be done on the pages provided. No additional scrap paper is to be used. Answers must be placed in the corresponding boxes in the answer column.

Name: _____

4A Find the following sum:

$$\begin{array}{r} 4\,3\,2\,1 \\ 5\,4\,3\,2 \\ 6\,5\,4\,3 \\ 7\,6\,5\,4 \\ 8\,7\,6\,5 \\ +\,9\,8\,7\,6 \\ \hline \end{array}$$

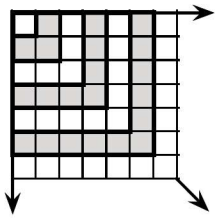
4B Jimmy, Kimmy, and Timmy are each thinking of a prime number. The product of Jimmy's and Kimmy's primes is 34. Kimmy's and Timmy's primes multiply to 85. What is the sum of their three prime numbers?

4C What is 1/2 of 2/3 of 3/4 of 2400?

Name: _____

Answer Column
4A
4B
4C
4D
4E
Do Not Write in this Space. *For PICO's Use Only.* *SCORE:*

4D Part of a 10 × 10 grid is shown in the following diagram. If the pattern shown is continued for the remaining grid, N percent of the entire figure will be shaded. Find the value of the whole number N. [Do not include the percent sign.]



– Page may be folded along dotted line. –

4E Use the following 3 equations to determine the value of the last expression.

□ + Δ + ● + ● = 15

□ + Δ + Δ + ● = 19

□ + □ + Δ + ● = 14

What is the value of: Δ + Δ + Δ + □ + □?

– Page may be folded along dotted line. –

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SOLUTIONS AND ANSWERS

4A 42591

4B 24

4C 600

4D 55

4E 25

4A *Strategy: Look for a pattern and use place value.*
The digits in the 1000s, 100s, 10s, and 1s column form a pattern. Each digit in the 1000s column is 1 greater than its corresponding digit in the 100s column, each digit in the 100s column is 1 greater than its corresponding digit in the 10s column, and each digit in the 10s column is 1 greater than its corresponding digit in the 1s column. Moving from right to left, column sums increase by 6. Since the sum in the ones column is 21, the remaining sums are 27, 33, and 39. The sum is: $39 \times 1000 + 33 \times 100 + 27 \times 10 + 21 \times 1 = 39000 + 3300 + 270 + 21 =$ **42,591**.

FOLLOW UP: Find the sum of the following numbers:
1234 + 2345 + 3456 + 4567 + 5678 + 6789 + 7890 + 8901 + 9012. [49,872]

4B **METHOD 1** *Strategy: Use properties of the products of even and odd numbers.*
The product of an even and an odd is even and the product of an odd and an odd is odd. Since the product of Jimmy and Kimmy's numbers is even and the only even prime is 2, either Jimmy or Kimmy must be thinking of the number 2. The product of Kimmy and Timmy's numbers is 85, so both primes must be odd. Therefore, it is Jimmy, not Kimmy, who is thinking of the number 2. Working backwards, Kimmy must be thinking of the number 17 and Timmy must be thinking of the number 5. The sum is therefore $2 + 5 + 17 =$ **24**.

METHOD 2 *Strategy: Use logic and properties of prime numbers.*
Use J, K, and T to represent the prime numbers that Jimmy, Kimmy, and Timmy are thinking about, respectively. It follows that $J \times K = 34$ and $K \times T = 85$.
Factoring 34 to primes gives $34 = 2 \times 17$ so either (J = 2 and K = 17) or (J = 17 and K = 2).
Factoring 85 to primes gives $85 = 5 \times 17$ so either (K = 5 and T = 17) or (K = 17 and T = 5). Since K = 17 in both situations, J = 2 and T = 5. The sum is 24.

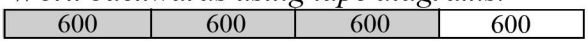
FOLLOW UPS: (1) *The product of 3 prime numbers is 4042. What is the least of the 3 primes?* [2] (2) *P and Q are primes such that P + Q = 28, and P < Q. Find the greatest possible value for prime Q.* [23]

4C **METHOD 1** *Strategy: The word "of" with fractions indicates multiplication.*
Multiply the fractions, expressing in lowest terms to simplify the computation:
$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times 2400 = \frac{6}{24} \times 2400 = \frac{1}{4} \times 2400 =$ **600**.

METHOD 2 *Strategy: Cancel shared factors of the numerators and denominators prior to multiplying.*
Express 2400 as a fraction and reduce fractions that equal 1.

$$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{2400}{1} = \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\overset{600}{\cancel{2400}}}{1} = 600$$

METHOD 3 *Strategy: Work backwards using tape diagrams.*

Calculate 3/4 of 2400. 

Calculate 2/3 of 1800. 

Calculate 1/2 of 1200. 

Thus, the answer is 600.

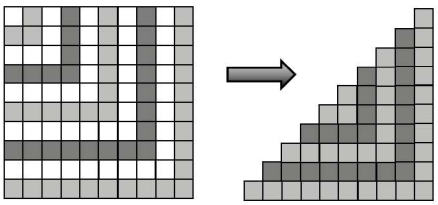
FOLLOW UPS: (1) *Express as a fraction in lowest terms:* $\frac{7}{8}\times\frac{8}{9}\times\frac{9}{10}\times\ldots\times\frac{2021}{2022}\times\frac{2022}{2023}$. $\left[\frac{1}{289}\right]$

(2) *Calculate* $\frac{2023!}{2022!}$ *where the symbol '!' is the factorial symbol.* [2023]

4D **METHOD 1** *Strategy: Find a pattern to count the shaded boxes.*

The shaded boxes appear in the even numbered rows. When the pattern is continued, the number of shaded boxes in the five even rows is: 3, 7, 11, 15, and 19. There are 10 rows altogether, with a total of $10 \times 10 = 100$ boxes. The fractional part of the diagram with shaded boxes is $(3 + 7 + 11 + 15 + 19)/100 = 55/100 = 55\%$, so N = **55**.

METHOD 2 *Strategy: Rearrange the shaded strips and use Gauss's method to find the sum.*



The number of shaded squares is the sum of 1 through 10, which can be arranged into 5 pairs of 11 by Gauss's method of summation: (1 + 10), (2 + 9), (3 + 8), (4 + 7), and (5 + 6). The total is $5 \times 11 = 55$ and there are 100 squares in the grid. The percent shaded is 55/100 = 55%, so N = 55.

FOLLOW UP: Extend the given diagram to a 100 × 100 grid. If the pattern shown is continued, to form the enlarged grid, what percent of the entire larger figure will be shaded? [50.5%]

4E **METHOD 1** *Strategy: Substitute letters for pictures and combine the equations.*

Let s = square, t = triangle, and c = circle.
The three equations are: $s + t + 2c = 15$, $s + 2t + c = 19$, and $2s + t + c = 14$.
The goal is to find the value of $3t + 2s$.
The sum of all three equations is $4s + 4t + 4c = 48$. Divide by 4 to discover that $s + t + c = 12$.
Subtract this equation from each of the original equations to find that c = 3, t = 7, and s = 2.
It follows that $3t + 2s = 3 \times 7 + 2 \times 2 =$ **25**.

METHOD 2 *Strategy: Play with adding and subtracting the equations.*

Add the second and third equations together and subtract the first equation from that sum:

$$\begin{array}{r} s + 2t + c = 19 \\ \underline{2s + t + c = 14} \\ 3s + 3t + 2c = 33 \end{array} \longrightarrow \begin{array}{r} 3s + 3t + 2c = 33 \\ \underline{-(s + t + 2c = 15)} \\ 2s + 2t = 18 \\ s + t = 9 \end{array} \longrightarrow \begin{array}{r} s + t + 2c = 15 \\ (s + t) + 2c = 15 \\ 9 + 2c = 15 \\ 2c = 6 \\ c = 3 \end{array} \longrightarrow \begin{array}{r} s + 2t + c = 19 \\ s + 2t + 3 = 19 \\ s + 2t = 16 \\ s + t = 9 \\ 2s + 3t = 25 \end{array}$$

FOLLOW UP: If $12A + 6B - C = 3791$ and $11A + 5B - C = 1768$, find the value of $A + B$. [2023]

NOTE: *Other FOLLOW-UP problems related to some of the above can be found in our four contest problem books and in "Creative Problem Solving in School Mathematics." Visit www.moems.org for details and to order.*