Factoring Stretch

1. The prime factorization of 315 is $3^2 \times 5 \times 7$. Since $3150 = 315 \times 10$, we only need to find the prime factorization of 10, which is 2×5. The prime factorization of 3150 is thus $\mathbf{2 \times 3^2 \times 5^2 \times 7}$.

2. The prime factorization of 180 is $2^2 \times 3^2 \times 5$. Divisors of 180 include numbers with the divisor 2 not used, used once or used twice (three possible ways). Similarly, the divisor 3 can be used one of three possible ways. The divisor 5 can be used or not used (two possible ways). Therefore, there are $3 \times 3 \times 2 =$ **18** total divisors.

3. Generalizing the solution in the previous example, to determine the total number of divisors of a number given its prime factorization, we add 1 to each of the exponents and then multiply the results. Thus, $x^3y^5z^2$ has $(3 + 1)(5 + 1)(2 + 1) = 4 \times 6 \times 3 =$ **72** divisors.

4. The prime factorization of 3150 is $2 \times 3^2 \times 5^2 \times 7$. The prime factorization of 180 is $2^2 \times 3^2 \times 5$. The least common multiple (LCM) of these two numbers is $2^2 \times 3^2 \times 5^2 \times 7 = 6300$. The greatest common divisor (GCD) of these two numbers is $2 \times 3^2 \times 5 = 90$. The first number is greater than the second by $6300 - 90 =$ **6210.**

5. The prime factorization of 12^3 is $2^6 \times 3^3$ and the prime factorization of 4^4 is 2^8. The greatest common divisor (GCD) of 12^3 and 4^4 is $2^6 = 64$ and the least common multiple is $2^8 \times 3^3 = 6912$. The sum of the GCD and LCM is $64 + 6912 =$ **6976**.

6. To factor $x^2 - 8x - 209$, we can start by looking for two integers that have a product of -209 and a sum of -8. Notice that $11 \times (-19) = -209$ and $11 + (-19) = -8$. That means we can write our polynomial in factored form as $(x + 11)(x - 19)$. The positive difference between these factors is $x + 11 - (x - 19) = 11 + 19 =$ **30**.

7. Setting our factor $x - 4 = 0$, we know that $x = 4$ is a root of the polynomial $x^2 + 13x - 12c + 4$. With that information, we know that substituting $x = 4$ into the polynomial should yield 0. We get $4^2 + 13(4) - 12c + 4 = 72 - 12c = 0$. Solving for c, we find $c =$ **6**.

8. To find the sum of the solutions of the equation $2x^2 - 11x = -12$, we first rewrite it in standard form as $2x^2 - 11x + 12 = 0$. For a quadratic equation $ax^2 + bx + c = 0$, the sum of the solutions is given by $-b/a$. Here, $a = 2$ and $b = -11$, so the sum of the solutions is $-(-11)/2 =$ **11/2**.

9. The quadratic polynomial is written in standard form, with $a = 1$, $b = -1$ and $c = -1$. Using the quadratic formula, the solutions of $x^2 - x + 1$ are $[1 \pm \sqrt{(1 + 4)}]/2 =$ $\mathbf{(1 \pm \sqrt{5})/2}$.

10. Using the quadratic formula, the solutions of $x^2 - 7x + 8$ are $[7 \pm \sqrt{(49 - 32)}]/2 = (7 \pm \sqrt{17})/2$. Therefore, $m = 7$, $n = 17$ and $p = 2$. The secret number is then $(17 - 7)/2 = 10/2 =$ **5**.