Sprint 11

The idea is that the ratio of the 121 catfish that were tagged and released on the first day compared to the unknown number of catfish in the lake c is proportional to the ratio of the tagged fish caught to the total fish caught on the second day, which is 22/48 = 11/24. We set up the proportion $121/c = 11/24$, and solve for c. The cross product is $11c = 121 \times 24$, so $c = (121 \times 24)/11 = 11 \times 24 = 264$. We would estimate that there are **264** catfish in the lake.

Sprint 12

Working backward, we subtract 3 and then divide by 3 to find each previous term. The third term is 39. So, the second term is $(39 - 3)/3 = 12$, and the first term is $(12 - 3)/3 =$ **3**.

Sprint 13

If the probability of choosing a blue marble from the bag is to be 1/2, then we must add enough blue marbles to make up half of the total number of marbles. Since there are 8 white and 6 red to start, we will need a total of 8 + 6 = 14 blue. There are 4 blue marbles to start with, so 14 – 4 = **10** blue marbles must be added to the bag.

Sprint 14

There is only 1 way to make a four-bead bracelet with just red beads. Likewise, there is only 1 way to make a bracelet with all green beads. There is only 1 way to make a bracelet with three red beads and one green. Similarly, there is 1 way to make a bracelet with three green beads and one red. When we consider using two red beads and two green beads, however, we find that there are 2 possible bracelets (R-R-G-G or R-G-R-G). Thus, there are 1 + 1 + 1 + 1 + 2 = **6** different color patterns.

Sprint 15

There are 12 possible doors to enter, and since we must leave by a different door, only 11 possible doors to exit by. Order counts, i.e. entering by door 1 and leaving by door 2 is different from entering by door 2 and leaving by door 1. So, there are 12 × 11 = **132** ways to do this.

Sprint 16

The only sums of 3 of these scores that total as much as 100 are 40 + 40 + 40 = 120, 40 + 40 + 29 = 109, 40 + 40 + 26 = 106 and 40 + 40 + 23 = 103, so Gina needs to shoot at least 4 arrows. Trial and error shows that Gina can get a total of 100 points with 4 arrows, such as 17 + 17 + 26 + 40 or 16 + 26 + 29 + 29. In fact, these are the only possibilities with 4 arrows. So, the fewest arrows Gina can shoot to score exactly 100 points is **4** arrows.

Sprint 17

Based on the 4-inch difference in their heights, Corbin and Addy could be either 60 and 64 inches tall, respectively, or 64 and 68 inches tall. But Beau, Corbin and Erin are the three shortest students. So, Corbin has to be 60 inches tall, and Addy is 64 inches tall. Since Dede is 1 inch shorter than Felix, he must be 68 inches tall, and Felix must be 69 inches tall. Beau is the shortest at 58 inches tall, so Erin must be 63 inches tall. The sum of Felix's and Erin's heights is 69 + 63 = **132** inches.

Sprint 18

If b is less than 20, then k can be 0, 1 or 2, making 0, 7 or 14 the resulting values of b, respectively. We don't need to worry about negative integers, since we're told that a is greater than 12 and a is less than b. This means that $12 < b < 20$. Considering this fact, we see that the only possible answer for b is 14, which is the result when $k =$ **2**.

Sprint 19

Let x be the cost of the ticket. Twenty students bought tickets at a cost of $20x$. The 10 additional students bought tickets at a cost of $10x$. This money was received as a refund of \$3.00 per student. We know 20 + 10 = 30 students, and 30 × \$3.00 = \$90, which was the total value of the refund and also was the cost of 10 tickets. Therefore, we can say $10x = 90$ and $x = 9$. So, the cost of each ticket was \$9, and \$9 × 20 = \$**180** was spent on beverages.

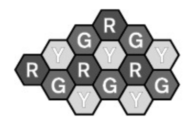
Sprint 20

Let the wider can have base radius r and height h. The other can is three times as tall with height $3h$ and base radius 12 inches. The formula for the volume of a cylinder is $V = \pi r^2 h$. Since the cans have the same volume, we have the equation $\pi r^2 h = \pi(12^2)(3h)$, which simplifies to $r^2 = (12^2)(3)$. Taking the square root of each side yields $r = 12\sqrt{3}$. Therefore, the wider can has radius $\mathbf{12\sqrt{3}}$ inches.

Sprint 21

Let b represent Shay's speed in still water, and let c represent the speed of the water's current. We know that Shay traveled 4 miles going downstream in 1 hour, with the current. Using the formula *distance = rate × time*, we can write an equation to represent her trip downstream: $4 = (b + c) \times 1$. When traveling upstream, against the current, Shay went the same distance but took 2 hours, which can be represented by the equation $4 = (b - c) \times 2$. This equation simplifies to $b - c = 2$. Subtracting the upstream equation from the downstream equation and solving for c gives us $(b - c) - (b + c) = (2 - 4)$, so $-2c = -2$ and $c =$ **1** mi/h.

Sprint 22

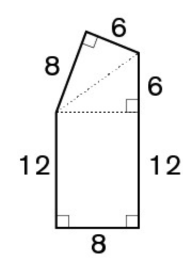


The figures show the **2** ways the figure can be colored red, yellow and green, so that no two hexagons with a common side are the same color.

Sprint 23

From the problem, we know there are 6 students who are both in eighth grade and wear glasses. Thus, in the chess club, there are 15 − 6 = 9 students who are eighth graders who do not wear glasses, and 9 − 6 = 3 students who wear glasses who are not in eighth grade. This is 18 students. Including the 8 students who are neither in eighth grade nor wear glasses, there are a total of 6 + 9 + 3 + 8 = **26** people in the chess club.

Sprint 24



Start by drawing a horizontal line parallel to the 8-inch base, as shown, dividing the original pentagon into a rectangle and a kite. The area of the rectangle is 12 × 8 = 96 in^2. Now, draw a segment joining the two non-right-angle vertices of the kite, and notice that the kite consists of two congruent right triangles. The area of the kite, then, is the sum of the areas of these two congruent right triangles, or 2 × (1/2)(8)(6) = 48 in^2. Therefore, the area of the pentagon is 96 + 48 = **144** in^2.

Sprint 25

At each of the eight corners of the original cube is a unit cube that has exactly three faces painted orange. In the center of each of the six faces of the original cube is a unit cube that has exactly one face painted orange. In the center of the original cube, there is a unit cube that has no faces painted orange. On each of the 12 edges of the original cube, there is a unit cube that has exactly two faces painted orange. Thus, the number of unit cubes with exactly two faces painted orange is **12** unit cubes.

Sprint 26

Based on the provided information, we can write the follow equations: [1] pop = 1.25(peep), [2] slug = 0.6(pop) and [3] slap = 2(slug). Using what we know from equation [2], we can substitute the value of a slug into equation [3] to get slap = 2 × 0.6(pop) = 1.2(pop). Then, using equation [1], we can calculate that 1.2(pop) = 1.2 × 1.25(peep) = 1.5(peep). Therefore, a slap is 1.5 × 100 = **150**% of a peep.

Sprint 27

Imagine the dance floor on the coordinate plane as a rectangle with vertices (0, 0), (72, 0), (72, 24) and (0, 24). Each unit square is a single tile, so we have 24 rows with 72 tiles each. The diagonal of this rectangle has slope 24/72 = 1/3. So, the diagonal passes through 3 tiles per row for a total of 3 × 24 = **72** tiles.

Sprint 28

Jeff's age has two digits, which we'll call x and y. We can represent Jonah's age with $10x + y$. Reversing this becomes $10y + x$, so $[(10y + x)/3] + 20 = 10x + y$. Multiplying both sides of this equation by 3, we get $10y + x + 60 = 30x + 3y$. Simplifying gives $7y + 60 = 29x$. Since x is a single digit, it must be large enough that $29x$ is greater than 60. Therefore, $x \geq 3$. If $x = 3$, then $29x = 87$ and $7y = 27$, which does not work. If $x = 4$, then $29x = 116$ and $7y = 56$, so $y = 8$. Therefore, Jeff must be $10x + y = 10 \times 4 + 8 = 40 + 8 =$ **48** years old.

Sprint 29

Starting at the M, we can go up, down, left or right to spell MATH 4 times, as seen in Figure 1. We can go up to the A, and then left or right to the T and H. This can also be done by going down to the A, and then left or right to the T and H, as seen in Figure 2, for a total of 4 paths. We can go up to the A, then right to the T, then up to the H (and the corresponding symmetrical paths) for 4 more paths, as seen in Figure 3. We can go up to the A and T and then left to the H (and the corresponding symmetrical paths) for 4 more paths, as seen in Figure 4. There are 4 additional paths shown in Figure 5, 4 more in Figure 6, and 4 more in Figure 7. This is a total of 7 × 4 = **28** paths that spell MATH. Alternatively, we can notice that the grid is symmetrical, so if we see there are 7 paths that spell MATH with the M and the A above it, and we know that there are four "MA" combinations, we can see that there are 4 × 7 = **28** paths that spell MATH.

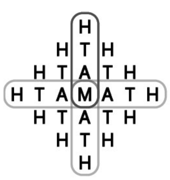


Figure 1

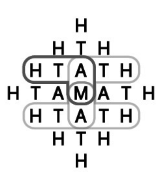


Figure 2

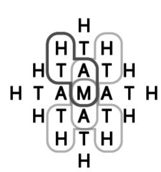


Figure 3

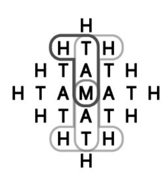


Figure 4

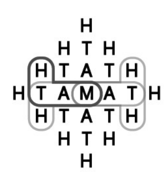


Figure 5

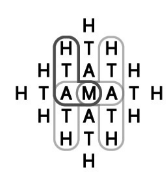


Figure 6

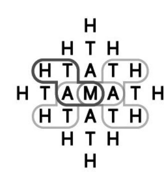


Figure 7

Sprint 30

Cross-multiplying, we can simplify the three equations to get [1] $4a = 3b$, [2] $9b = 8c$, and [3] $3c = 2d$. Let's rewrite equation [3] as $c = (2/3)d$. Then, we can substitute this value for c into equation [2] to get $9b = 8(2/3)d \rightarrow 9b = (16/3)d \rightarrow (27/16)b = d$. We can rewrite equation [1] to be $a = (3/4)b$. Therefore, $ad/b^2 = [(3/4)b \times (27/16)b]/b^2 = [(81/64)b^2]/b^2 =$ **81/64**.