Warm-Up 3

51. The prime factorization of 2000 is $2^4 \times 5^3$. There are $(4 + 1)(3 + 1) = 5 \times 4 = 20$ total positive divisors of 2000. To specify a perfect square divisor, we must choose even exponents for each prime factor. For the factor of 2, we can choose an exponent of 0, 2 or 4 (3 options). For the factor of 5, we can choose 0 or 2 (2 options). This gives us $3 \times 2 = 6$ perfect square divisors. Therefore, the percent of divisors that are perfect squares is 6/20 = 3/10 = 30/100 = **30**%. Alternatively, we can list the divisors of 2000, as shown in the array to the right. The 6 perfect square divisors of 2000 are 1, 4, 16, 25, 100 and 400. These 6 perfect squares represent **30**% of the 20 divisors.

1	5	**25**	125
2	10	50	250
4	20	**100**	500
8	40	200	1000
16	80	**400**	2000

52. If all writers work at the same rate, then the 822 questions written by 6 writers in one year means that each writer wrote $822 \div 6 = 137$ questions in that one year. In four years, one writer can write $137 \times 4 =$ **548** questions.

53. There are 5 ways to give 2 pumps of the same flavor and then "five choose two" = $(5 \times 4) \div 2 = 10$ ways to pick two different flavors. Then there are $3 + 1 = 4$ possible toppings, including no topping. That makes $(5 + 10) \times 4 = 15 \times 4 =$ **60** combinations of shaved ice.

54. If we subtract $3x + 6$ from both sides of the first equation, we get $2x = 4$, to which the solution is $x = 2$. Substituting this value into the second equation, we get $a(2) - 4 = 0$, which simplifies to $2a = 4$ and then $a =$ **2**.

55. Suppose Jimmy drove at 35 mi/h for h hours. Then he covered $65 + 35h$ miles in $(h + 1)$ hours, so $65 + 35h = 45(h + 1)$. Expanding and subtracting $35h + 45$ from both sides gives $20 = 10h$, so $h = 2$. We can calculate the total distance Jimmy drove in two ways: $45 \times (2 + 1) = 45 \times 3 =$ **135** miles *or* $35 \times 2 + 65 \times 1 = 70 + 65 =$ **135** miles.

56. There are 27 intervals from 0 to 3 on this number line, which is 9 intervals per unit. Since $1/9 \times 9 = 1$, the letter at 1/9 is the first letter, which is A. Since $2\ 2/3 \times 9 = 8/3 \times 9 = 24$, the next letter is the 24th letter, which is X. Likewise, the letter at 1 is the $1 \times 9 =$ 9th letter, which is I, and the letter at 2 1/9, is the $2\ 1/9 \times 9 = 19/9 \times 9 =$ 19th letter, which is S. The math word is **AXIS**.

57. To find the units digit of the expression, we can add just the units digit of each term. The first term 2 has a units digit of 2. The second term is $2^0 = 1$, with a units digit 1. The third term $2^{0+2} = 2^2 = 4$ has a units digit 4. The fourth term $2^{0+2+5} = 2^7 = 128$ has a units digit 8. Adding the units digits of each term gives $2 + 1 + 4 + 8 = 15$, so the units digit of the sum is **5**.

58. The numbers at the far right of each row of Geo's arrangement are known as the triangular numbers. The nth triangular number can be calculated as $n(n + 1) \div 2$, so the last number of the sixth row is $6(6 + 1) \div 2 = 21$ and the last number of the seventh row is $7(7 + 1) \div 2 = 28$. Knowing this, we can say that the sum of the first and last numbers of the seventh row is $22 + 28 =$ **50**.

59. To evaluate $\sqrt{(20\sqrt{25})}$, we start with the innermost radical sign: $\sqrt{25} = 5$. Now we have $\sqrt{(20 \times 5)} = \sqrt{100} =$ **10**.

60. The terms of an arithmetic progression must have a common difference, which we will call d. The three angles are 35, $35 + d$ and $35 + 2d$, and their sum must be 180 degrees. We need to solve the equation $35 + 35 + d + 35 + 2d = 180$. This simplifies to $105 + 3d = 180$, which simplifies further to $3d = 75$ and finally $d = 25$. The three angles are 35, 60 and 85, with the measure of the largest angle being **85** degrees.