Warm-Up 1

31. If Elmina rounds each number to the nearest thousand before adding, she should get a sum of 5000 + 32,000 + 1000 = **38,000**.

32. Since we know that the sum of the first six terms of the arithmetic sequence is 42, we can calculate that the average of the terms is 42 ÷ 6 = 7. Given that the first term is 2, this is enough to work out that the six terms must be 2, 4, 6, 8, 10, 12, where the average of 7 is half-way between the 6 and the **8**. Alternatively, some mathletes may know that the sum of an arithmetic sequence is the average of the first and last terms times the number of terms. Since we know that the average is 7, we can calculate that the sum of the first and last terms must be 2 × 7 = 14 and the last term must be 14 − 2 = 12. The common difference must be added five times to get from the first to the sixth term, so that difference must be (12 − 2) ÷ 5 = 10 ÷ 5 = 2. Now we write out the six terms 2, 4, 6, 8, 10, 12, and we see that the fourth term is **8**.

33. We can divide 137 by 26 to find that the quotient is 5 and the remainder is 7. For this problem, we are more interested in the remainder, because the 137th letter on the banner is simply the 7th letter of the alphabet, which is **G**.

34. If $x + 3 = 10$, then $x = 7$. The value of $x^2 + 3^2$ is $7^2 + 3^2 = 49 + 9 =$ **58**.

35. There are 4 quarts in 1 gallon, so there are 2 quarts remaining after Nadine removed 2 quarts. There are 2 pints in 1 quart, so those 2 quarts are 4 pints. Nadine added 3 pints to this, so that makes 7 pints. There are 2 cups in 1 pint, so those 7 pints are 14 cups. Nadine removed 4 cups, so she ends up with 10 cups, or **80** fluid ounces, since there are 128/16 = 8 fluid ounces in a cup. We could also convert all quantities to ounces at the start, in which case we could compute 128 − 64 + 48 − 32 = **80** fluid ounces.

36. Alice chooses a number A and Maud chooses a number M, both from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, with $M \neq A$, as was confirmed up front by Nina. Alice can know for sure that AM is even if and only if her number A is even. Then, with A advertised to be even, Maud can know for sure that $A + M$ is even if and only if her number M is even. At this stage, A is an element of {2, 4, 6, 8, 10} and M is an element of {2, 4, 6, 8, 10}. If $A = 2$, then M being even and greater than 2 means that A is a proper divisor of M; if $A = 4$, then M could be 8 as far as Alice knows, making A a proper divisor of M. Such an issue does not occur for $A \geq 6$, so Alice can validly make such an assertion about A not being a proper divisor of M if and only if A is 6, 8 or 10. The remaining candidate (A, M) pairs are (6, 2), (6, 4), (6, 8), (6, 10), (8, 2), (8, 4), (8, 6), (8, 10), (10, 2), (10, 4), (10, 6) and (10, 8). Now we deal with the units digit of A^2 not equaling the units digit of M. Of our remaining candidate ordered pairs, only (8, 4) violates this congruence relation. Therefore, Maud can validly make her assertion if and only if $M \neq 4$, eliminating 3 ordered pairs from being candidates, leaving (6, 2), (6, 8), (6, 10), (8, 2), (8, 6), (8, 10), (10, 2), (10, 6) and (10, 8). Alice's assertion about the greatest common divisor provides no new information for eliminating any of the currently remaining 9 candidate ordered pairs, because $\gcd(A, M) = 2$ for all 9 ordered pairs. This step is useless. Every remaining candidate pair with $M \leq 6$ has $A \geq 8$, so Alice's number is definitely greater; every remaining ordered pair with $M = 10$ has $A \leq 8$, so Alice's number is definitely not greater. The only value that would cause Maud to have uncertainty is $M = 8$, for which A could be 6 (less than M) or 10 (greater than M). Alice's response that $A \ngeq M = 8$ requires $A = 6$, so Alice's integer must be **6**.

37. Since 4 times 3/4 is 3, Alex will need to quadruple his recipe. He will need 4/2 cups of water and 4/3 cups of milk. Converting both of these quantities to sixths, we see that Alex will need 12/6 cups of water and 8/6 cups of milk, which is 4/6 or **2/3** more cups of water than milk.

38. If we look at this formation from each of the four sides, we can see a total of 6 units per side, so the perimeter of the figure is 4 × 6 = **24** units.

39. Joe ate 3/2 × 1/6 = 3/12 = **1/4** of the pie.

40. To find the units digit of the sum $1^2 + 2^2 + 3^2 + 4^2 + 5^2$, we take the units digits of the squares and add them: 1 + 4 + 9 + 6 + 5 = 25, which has a units digit **5**.