Workout 1

141. There are $100 \div 8 = 12.5$ cups in 100 fluid ounces, so there are $12.5 \times 30 = \mathbf{375}$ calories in a full carton of almond milk.

142. There are 4 single-digit positive even integers, namely 2, 4, 6 and 8. From 10 to 99, there are $99/2 - (10 - 1)/2 = 99/2 - 9/2 = 90/2 = 45$ two-digit positive even integers. From 100 to 999, there are $999/2 - (100 - 1)/2 = 999/2 - 99/2 = 900/2 = 450$ three-digit positive even integers. That's 499 positive even integers so far, so we just need the 1 four-digit number 1000. The number of digits used to write these 500 positive even integers is $4 \times 1 + 45 \times 2 + 450 \times 3 + 1 \times 4 = 4 + 90 + 1350 + 4 = \mathbf{1448}$ digits.

143. For the square photo, including the 1-inch frame on all sides, the outer dimensions are $4 + 2 = 6$ in by $4 + 2 = 6$ in. The total area of the photo with the frame is $6 \times 6 = 36$ in^2, so the area of the frame is $36 - 16 = 20$ in^2. For the rectangular photo, the outer dimensions are $2 + 2 = 4$ in by $8 + 2 = 10$ in. The total area of the photo with the frame is $4 \times 10 = 40$ in^2, so the area of the frame is $40 - 16 = 24$ in^2. That means the absolute difference between the areas of the two frames is $|24 - 20| = \mathbf{4}$ in^2.

144. We can solve this problem algebraically as shown below, but we can also solve it with some proportional reasoning. Let's say the original rate of speed is r mi/h and the distance is d miles. At rate r, it takes 3 hours (9/3 hours) to go the distance d. At rate $(r + 10)$, it takes 2 hours 40 minutes (8/3 hours) to go the same distance. If it takes 8/9 of the time to go the same distance, then the car must be traveling 9/8 as fast. This means that the extra 10 mi/h is the 1/8 of r, so r must be $8 \times 10 = 80$ mi/h. Traveling for 3 hours at 80 mi/h is **240** miles, so this is the distance between Sprintville and Countdown City.

$$3r = (3 - 1/3)(r + 10)$$
$$9r = 8r + 80$$
$$r = 80$$

145. Since the sequence of primes is arithmetic, the middle number must be $111 \div 3 = 37$. Now we need to find two primes equidistant from 37, one less and one more. The numbers 31 and 43 are both prime and both 6 away, so the sequence could be 31, 37, 43. Other candidates are 13, 37, 61 or 7, 37, 67 or 3, 37, 71. The last of these has the least possible product, which is **7881**.

146. The value of 5! is 120 and its divisors are shown in the array to the right. Twelve of these divisors are also elements of the set {1, 2, 3, …, 25}, so the probability is **12/25**.

1	**3**	**5**	**15**
2	**6**	**10**	30
4	**12**	**20**	60
8	**24**	40	120

147. To find the sum of all positive palindromes less than 2025 that are divisible by 99, we first note that the positive multiples of 99 less than 2025 are 99×1 through 99×20, which gives the numbers 99, 198, 297, 396, 495, 594, 693, 792, 891, 990, 1089, 1188, 1287, 1386, 1485, 1584, 1683, 1782, 1881 and 1980. Scanning the list of 20 values, we see that only 99 and 1881 are palindromes. Their sum is $99 + 1881 = \mathbf{1980}$.

148. Since the average height of a professional basketball player is 78 inches and there are typically 15 players per team, the combined height of the players on a basketball team is $15 \times 78 = 1170$ inches. Since the average height of a professional baseball player is 74 inches and there are typically 26 players per team, the combined height of the players on a baseball team is $26 \times 74 = 1924$ inches. The absolute difference between these two totals is $|1924 - 1170| = \mathbf{754}$ inches.

149. The divisors of 18 are the odd numbers 1, 3, 9 and the even numbers 2, 6, 18. Only the evens are not divisors of 63, so the answer is **3** integers.

150. The first six terms of the sequence are 4, 7, 13, 25, 49, 97. The answer is **97**.