Sprint 1

$3^4 - 2 \times 4^2 = 81 - 2 \times 16$ [Exponentiation first.]
$= 81 - 32$ [Multiplication next.]
$= \mathbf{49}$. [Subtraction last.]

Sprint 2

$$\begin{array}{r} 4\,326\,052 \\ -\underline{4\,325\,131} \\ \mathbf{921} \end{array}$$

Sprint 3

Reordering the set elements in increasing order yields {0, 3, 4, 7, 8, 11, 16}. Since we have an odd number of elements, the median is the middle element of this ordered set. In the case of this set, the middle element is the fourth element, which is **7**.

Sprint 4

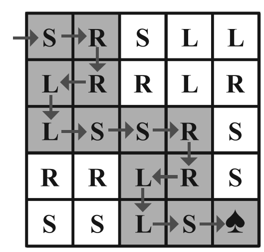
Parker got 11. Each friend got twice that: $2 \times 11 = 22$. There are 3 friends, so the total count is $11 + 3 \times 22 = 11 + 66 = \mathbf{77}$ pieces.

Sprint 5

The iterative steps 3 and 4 yield what is called the digits-sum of the original number. Summing the digits of a number and taking the remainder upon dividing the sum of the digits by 9 gives the remainder upon dividing the original number by 9. The only nonnegative integer that yields a digit-sum of 0 is 0; a positive integer that is divisible by 9 has a digit-sum of 9, instead of the actual remainder value 0. The sum of the digits of 135 is 9, so its digit-sum is 9, indicating 135 to be divisible by 9. If we multiply a positive multiple of 9 by *any* positive integer (it does not have to be two-digit), the result is also a multiple of 9, so the digit-sum is **9**.

Sprint 6

Let's trace through the rooms, starting with the arrow entering the room at the upper left, shading each room that we enter, and drawing an arrow into the next room in accordance with the L, R, or S in the room we just entered. Once we reach the spade room, we count the number of shaded rooms, which is **13** rooms.



Sprint 7

$n + 18 = 4 \times 5 = 20$, so $n = 20 - 18 = \mathbf{2}$.

Sprint 8

We need an integer that is divisible by both 2 and 3, thus by 6, and it must be greater than 100. Since $100 \div 6 = 16$ R 4, the next multiple of 6 up from 100 is $17 \times 6 = 102$. So, the fewest number of students who could have signed up is **102** students.

Sprint 9

Rectangular prisms have 6 faces. For any 1 face, there are 4 adjacent faces that must not be colored the same as the first face, and only the 1 remaining face is non-adjacent and permitted to be the same color. Therefore, at most 2 faces (opposite faces) are allowed to be the same color, so at least $6/2 = 3$ colors are needed. There are 3 pairs of opposite sides, so the minimum number of colors needed is **3** colors.

Sprint 10

Let l be the length of the rectangle. Then the width $w = l + 1$. We need the perimeter, which is $2(l + w) = 2(l + l + 1) = 4l + 2$. We are given that 72 ft^2 is the enclosed area lw. Therefore, $72 = lw = l(l + 1) = l^2 + 1l$. Rearranging yields: $0 = l^2 + 1l - 72 = (l + 9)(l - 8)$, so $l = -9$ feet or $l = 8$ feet. It makes no sense for the length of a side to be negative, so we must have $l = 8$ feet and perimeter $4 \times 8\text{ ft} + 2\text{ ft} = \mathbf{34}$ feet.

Sprint 11

Because there are $\frac{3}{8}$ as many heads as there are legs, there are $\frac{8}{3}$ as many legs as heads, so the average number of legs per head is $2\frac{2}{3}$, which is $\frac{2\frac{2}{3} - 2}{4 - 2} = \frac{2/3}{2} = \frac{1}{3}$ of the way from 2 [# legs per chicken] to 4 [# legs per goat]. Therefore, $\frac{1}{3}$ of the animals are goats, and $\frac{2}{3}$ are chickens, so the goat-to-chicken ratio is 1:2. Thus, the minimum number of animals is $1 + 2 = 3$ animals, meaning there is 1 goat and **2** chickens.

Sprint 12

The two highest scores are 8.5 and one of the two 8.0 scores, so these are discarded; the two lowest scores are 7.0 and 7.0 (both of the two 7.0 scores), so these are discarded. The scores that are used are the one remaining 8.0 and the two scores of 7.5. Therefore, the point total for the dive is $3.5(8.0 + 7.5 + 7.5) = 3.5(23) = 3(23) + \frac{1}{2}(23) = 69 + 11.5 = \mathbf{80.5}$ points.

Sprint 13

$1\frac{3}{4} = \frac{7}{4}$ cups are required for 24 cookies, so for 18 cookies: $\frac{18}{24}\left(\frac{7}{4}\right) = \frac{3}{4}\left(\frac{7}{4}\right) = \frac{21}{16} = \mathbf{1\frac{5}{16}}$ cups.

Sprint 14

With a rectangle, two vertices may be the endpoints of a short side, of a long side, or of a diagonal. Given two such lengths, the shorter must be a side, while the longer may be a side or a diagonal. If the 5 meters is a diagonal, then we have a 3-4-5 right triangle and the unknown length is the second side, which must be 4 meters. If the 5 meters is the longer side, the unknown length is the diagonal, even longer than the 5 meters. Thus, the minimum possible distance in question is **4** meters.

Sprint 15

Of the 64×144 calculators in the lot, 64×12 were tested. Therefore, the fraction tested is $\frac{\cancel{64\times}12}{\cancel{64\times}144} = \frac{1}{12}$. If 12 times as many calculators are tested, we expect 12 times as many failures, thus $12 \times 2 = \mathbf{24}$ calculators.

Sprint 16

This problem involves a weighted average with the \$30 value weighted as $75\% = \frac{3}{4}$ and the \$10 value weighted as $25\% = \frac{1}{4}$. So, $\frac{3}{4}(30) + \frac{1}{4}(10) = \frac{90+10}{4} = \frac{100}{4} = \$\mathbf{25}$ or \$**25.00**.

Sprint 17

$$\left(11 + \frac{1}{2}\right) \times \left(12 + \frac{5}{12}\right) = \left(11 \times 12 + 11 \times \frac{5}{12} + \frac{1}{2} \times 12 + \frac{1}{2} \times \frac{5}{12}\right) = \left(132 + \frac{55}{12} + 6 + \frac{5}{24}\right) =$$
$$\left(132 + 6 + \frac{2\times55 + 5}{24}\right) = \left(138 + \frac{115}{24}\right) = 142\frac{19}{24}\text{ ft}^2\text{, which rounds to } \mathbf{143}\text{ ft}^2.$$

Sprint 18
The length l and width w are related as $w = l - 50$. Therefore, the perimeter is given by $500 = 2(l + w) = 2(l + l - 50) = 4l - 100$, so $l = \frac{500+100}{4} = 150$ ft. The enclosed area is given by $lw = l(l - 50) = 150 \times 100 = \mathbf{15,000}\ \text{ft}^2$.

Sprint 19
We need to determine the estimated arrival time down to the minute, so we need to determine the remaining travel time, distance divided by speed, in terms of minutes: $\frac{60\ \cancel{\text{mi}}}{75\frac{\cancel{\text{mi}}}{\cancel{\text{hour}}}} \times \frac{60\ \text{minutes}}{\cancel{1\ \text{hour}}} = \frac{4}{5} \times 60\ \text{minutes} =$ 48 minutes, which is 12 minutes short of 1 hour. Therefore, starting at 10:32, add 1 hour to get 11:32, and then subtract 12 minutes to result in **11:20** a.m.

Sprint 20
Based on the inforamtion given, we have $x^5 = \frac{2}{3}x^4$, so $x^4\left(x - \frac{2}{3}\right) = 0$. Therefore, $x = 0$ or $x = \frac{2}{3}$, but it must be the latter value because 0 does not satisfy the positivity requirement. Thus, $\frac{x^{10}}{x^8} = x^2 = \left(\frac{2}{3}\right)^2 = \mathbf{\frac{4}{9}}$.

Sprint 21
If the two bread slices are selected to be the same, then there are 3 flavor choices. If they are selected to be different, then there are ${}_3C_2 = \frac{3!}{2!\,1!} = 3$ choices, totaling 6 choices for the bread. For each of these, there are 3 choices of filling (ham only, cheese only, both). So, James can make $6 \times 3 = \mathbf{18}$ different sandwiches.

Sprint 22
Let N be the total number of students in Mr. Short's homeroom. Then working backward, we see that $6 = \frac{1}{4}\left(1 - \frac{1}{3}\right)N = \frac{1}{4}\left(\frac{2}{3}\right)N = \frac{1}{6}N$, so $N = 6 \times 6 = \mathbf{36}$ students.

Sprint 23
We need meters per second, not centimeters per second, so convert the stride of 140 cm to meters: $140\ \cancel{\text{cm}} \times \frac{1\ \text{m}}{100\ \cancel{\text{cm}}} = 1.4$ meters. The number of pulses going above the dashed line for the 3 g threshold to count as strides is 19. Therefore, a total assumed distance of 19×1.4 meters [do *not* calculate that yet] is covered in 20 seconds, making the average speed $\frac{19 \times 1.4}{20} = 19 \times 0.07 = \mathbf{1.33}$ m/s. [Notice that waiting so we could do the very easy division first made for a simpler multiplication later, which can be important for the Sprint Round as a speed round without support of electronic calculators.]

Sprint 24
The triangle inequality property tells us $3 = 5 - 2 < x < 5 + 2 = 7$, so x as an integer must be 4, 5 or 6. The median is the middle value, **5**.

Sprint 25
\$150 each year averages to $\frac{\$150/\cancel{\text{yr}}}{12\frac{\text{mo}}{\cancel{\text{yr}}}} = \frac{\cancel{6\times}\$25}{\cancel{6}\times 2\ \text{mo}} = \$12.50/\text{mo}$. So, $\$5.95N > \$12.50/\text{mo} + \$3.95N$, where N is the number of movies rented per month. Thus, $\$2N > \$12.50/\text{mo}$, making $N > \frac{\cancel{\$}12.50/\text{mo}}{\cancel{\$}2} = 6.25/\text{mo}$. The least such integer is **7** movies per month.

Sprint 26

Note that $345{,}600 = 3456 \times (2 \times 5)^2$. The sum of the digits of 3456 is 18, which is divisible by 9, so 3456 is likewise divisible by $9 = 3^2$, leaving a quotient of 384. The sum of the digits of 384 is divible by 3 but not by 9, so that is the case with $384 = 3 \times 128 = 3 \times 2^7$. Combining all this yields $345{,}600 = 2^9 \times 3^3 \times 5^2 = 2^6 \times (2^3 \times 3^3) \times 5^2 = 4^{6/2} \times 6^3 \times 5^2 = 6^3 \times 5^2 \times 4^3$. The product of the exponents is $abc = 3 \times 2 \times 3 = \mathbf{18}$.

Sprint 27

Of 12 flowers, 6 are required to be orchids, leaving flexibility for the remaining 6 to be any mix of 4 kinds of flowers, including possibly more orchids. Let's use what I call the bars and blanks method. [A variety of names are given to this method.] We have 12 blanks, each representing one flower, with O (for orchid) in the last 6 since we require a minimum of 6 orchids; we have 3 bars to intersperse among the empty blanks to separate the roses from the lilies, the lilies from the violets, and the violets from the extra orchids. For example: | ___ ___ ___ ___ || ___ ___ _O_ _O_ _O_ _O_ _O_ _O_ would indicate 0 roses (because of 0 blanks to the left of the first bar), 4 lilies (because of 4 blanks between the first and second bars), 0 violets (because of 0 blanks between the second and third bars) and 2 extra orchids (because of two empty blanks to the right of the last bar, for a total of 8 orchids). The number of possible distinct bouquets is the number of orderings of the 3 bars and the 6 empty blanks, which is ${}_{6+3}C_6 = \frac{9!}{6!\,3!} = \frac{9\times8\times7}{3\times2\times1} = \frac{72\times7}{6} = 12 \times 7 = 84$. So, that's **84** groups of a dozen flowers.

Sprint 28

Cross-multiplying yields $(x + 2)(y + 3) = 6$. The only ways to factor 6 into the product of two integers is $(1)(6)$, $(2)(3)$, $(-1)(-6)$ and $(-2)(-3)$—each of which can go in either order as to which factor is $(x + 2)$ and which is $(y + 3)$. The 8 factors for $(x + 2)$ occur in 4 pairs, each pair having one positive value and one counterpart equal-magnitude negative value, with the sums canceling each other to 0, so the sum of all eight $(x + 2)$ factors is 0. To get the sum of all eight corresponding x values, subtract 2 for each of the 8 factors to end up with $0 - 8 \times 2 = \mathbf{-16}$.

Sprint 29

$a_0 = 4$;

$a_1 = a_1$; [an as yet unknown value]

$a_2 = a_1 + 2a_0 = a_1 + 8$;

$a_3 = a_2 + 2a_1 = a_1 + 8 + 2a_1 = 3a_1 + 8$;

$a_4 = a_3 + 2a_2 = 3a_1 + 8 + 2(a_1 + 8) = 5a_1 + 24 = 26$, so $a_1 = \frac{26-24}{5} = \frac{2}{5}$ and $a_3 = \frac{6}{5} + 8 = \frac{46}{5}$;

$a_5 = a_4 + 2a_3 = 26 + 2 \times \frac{46}{5} = \frac{130+92}{5} = \mathbf{\frac{222}{5}}$.

Sprint 30

Each of the 8 vertex blocks has 3 sides painted, thus probability $\frac{1}{2}$ coming up unpainted. The 12 edge blocks except vertex blocks total $12 \times 8 = 96$ blocks, each having 2 sides painted, thus probability $\frac{2}{3}$ coming up unpainted. The 6 face blocks except the edge and vertex blocks total $6 \times 8 \times 8 = 384$ blocks, each having 1 side painted, thus probability $\frac{5}{6}$ coming up unpainted. Each of the remaining blocks, the interior $8 \times 8 \times 8 = 512$ blocks, has 0 sides painted, thus probability 1 coming up unpainted. Therefore, the overall probability of all blocks rolling with the upward facing side being unpainted is given by $\left(\frac{1}{2}\right)^8 \left(\frac{2}{3}\right)^{96} \left(\frac{5}{6}\right)^{384} 1^{512} = \frac{2^{96}\times5^{384}}{2^8\times3^{96}\times2^{384}\times3^{384}} = 2^{-296} \times 3^{-480} \times 5^{384}$, so the desired answer is the sum of the exponents, $-296 - 480 + 384 = \mathbf{-392}$.